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ABSTRACT

The Comprehensive School Mathematics Program (CSMP) materials presented here were in the process of development at the time of publication. It is noted that children generally classified as slow learners are, in many cases, pupils who have had unhappy experiences with learning. The instructional approach is designed to: (1) inspire pupils with a spirit of enthusiasm for mathematics and a curiosity about numbers and mathematical processes; (2) enrich student notions of numbers; and (3) give pupils a chance to make their own discoveries about numbers. There is a conviction expressed that paper and pencil should be banned in work which is done with slow learners in small groups. The use of calculators is promoted as a substitute. Activities presented are collected under: (1) Hand-Calculator; (2) Minicomputer Games; and (3) Detective Stories. (MP)

Comprehensive School Mathematics Program
CEMREL, Inc.
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CSMP Mathematics for the Intermediate Grades

Math Play Therapy

Volume I

CEMREL, Inc.
Comprehensive School Mathematics Program
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The Comprehensive School Mathematics Program materials included herein are in the process of development. As a part of our continuing effort to evaluate and improve them, we ask that you comment in detail on the materials and on the way in which you used them.

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Comprehensive School Mathematics Program

Math Play Therapy

CEMREL, Inc.

NOTE ON AUTHORSHIP

"Math Play Therapy" has been produced in answer to requests from many teachers in our pilot test classrooms. It is the result of one year of experimental work with students who were at the fourth grade level and new to CSMP. We hope this book will be helpful to the teachers of fourth grade; perhaps the activities described herein are also appropriate for third grade children who are veterans of CSMP. It is a first draft of a supplementary text which we hope will later include fifth and sixth grade levels.

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TABLE OF CONTENTS

INTRODUCTION

We have the firm conviction that children who have learning difficulties can, nevertheless, become involved in every strand of CSMP with some supplementary help. Our experience with such children leads us to recommend small group activities on a weekly or bi-weekly basis, in addition to the regular class period. In order to adapt materials for these children, we have worked with two or three groups from each of our two development classes, four students to a group, for one half-hour each week. The children have looked forward to these special sessions and have shown much progress, both in the small groups and in the regular class. We believe it is a very positive situation. We understand, however, that implementation of this suggestion will depend on a great many variables relative to each individual teacher. So we stress that this is not a rigid requirement, but a recommendation which can be followed in the way that works best in a particular situation.

It is our observation that children who are generally classified as "slow learners"[◇] are, in many cases, children who have had unhappy experiences with learning (in general, or in a specific area) and who have made a personal decision that they are unable to learn. Quite often we find that these children are also quite creative and, consequently do not easily fit into the pattern of behavior which we may have in our minds for them, nor achieve well in such a pattern.

[◇] We feel this is not the most appropriate term; furthermore, we do not like its negative connotations, and in fact we do not ever like to label people. However, being in the unhappy position of needing to find some method of defining a particular group of children in order to talk about them at all, we reluntantly use this terminology.

Others among them may have great difficulty in focusing on a particular situation, being very easily distracted. They often seem to be subject to an inner turmoil which keeps them from being able to relax and deal with the situation at hand. This is why we work with them in a small, closed room. In addition, these children may have widely varying temperaments and abilities to concentrate and to relate to other people and to the situation on any two specific days. With this in mind, we suggest that it is important to consider which children have temperaments that are complementary, rather than too similar, when deciding the makeup of a particular group.

As we have worked within the traditional classroom format and simultaneously with small groups of slow learners, we have been evolving and clarifying a pedagogy for the children who have difficulty in learning by the traditional methods and who are frequently "lost by the wayside". It is our premise that they need <u>not</u> be lost to mathematics and our experiences in the small groups reinforce this belief.

This pedagogy is firmly grounded in our three specific goals which are to

- inspire in these students an enthusiasm for mathematics (that which is not form, but spirit) and a curiosity about numbers and mathematical processes.
- enrich their notion of numbers.
- give them a chance to make their <u>own</u> discoveries about numbers.

While striving to accomplish these goals, we have made some observations which we believe are generally true.

1) Such students are at a distinct disadvantage in the regular classroom situation because its rather rigid structure is, for most of

them, the same as a cage is f r any creature accustomed to an environment with much space and freedom of movement.

2) Many of these children are most at home in their bodies, not in their minds, and their natural mode of communication is more likely to be oral and physical than verbal. Thus, paper and pencil exercises hold almost <u>no</u> appeal, give slight motivation and command very little attention. Games, on the other hand, not only have a great deal of appeal for most children but allow oral and tactile responses, as well as movement around the room. They involve these students with the material to be learned in the ways which are most natural to them and which help develop their understanding and increase their ability to concentrate. It is for these reasons that we chose games as our usual *modus operandi* in the small groups.

3) Slow learners need a strong support for their numerical thinking, and our experience at the present time leads us to the conclusion that the Papy Minicomputer[◇] is the best pedagogical aid we have seen for providing such support. Our observation of the children clearly indicates that they happily accept it as a friend and an ally in their endeavor to expand their knowledge of numbers. Because it is, by its very nature, both a game which allows them freedom and physical involvement and a creative and open situation which is always inviting to children, they almost invariably respond positively to the activities which make use of it.

It is equally clear that for those who have learning difficulties, the process of becoming fluent in the mathematical language of the Minicomputer is a very slow one — one which requires a great deal of patience on the part of the teacher, as well as a strong foundation of faith (in both her or his ability to teach and the students' ability to learn) and the forbearance to wait several months to evaluate the learning which has taken place.

[◇]Readers unfamiliar with the Minicomputer are referred to the Appendix to Chapter 2 on page 58.

4) Slow learners often need appealing tools which can substitute for paper and pencil and which will hold their attention. In our work, we have made essential use of the hand-calculator and would like to expand on the observations we have made as we used it with the students and the reasons we are now convinced of its value.

We cannot overemphasize our conviction that paper and pencil should be considered "off limits", "illegal", and "verboten" in the work which is done with slow learners in the small groups. For many such children, a most dramatic symbol of their failure resides in these two objects. We want them to be free of such symbolization in order to explore their potential to understand mathematics with new eyes. Therefore, our first reason for using the hand-calculator is to give them a substitute for paper and pencil, one which still will allow them to record the mathematical calculations as they occur. Since the calculator makes it possible for them to do this individually, the group is not limited to one oral response to a question or a calculation. Rather, each child can respond at his or her own speed. The teacher can silently check each display, giving those who need more time a better opportunity to make individual discoveries.

A second reason for using the hand-calculator is its attractiveness to the students. Because it is so fascinating to them, it serves as a motivation and as an aid in developing their ability to concentrate as they use it from week to week. Because it is clearly a tool used by adults and by those who are more advanced in mathematics, it gives them the strong psychological message that, instead of being looked upon as babies, they are considered mature enough to understand and manipulate such a marvelous — even magical — object.

Perhaps the most effective, as well as the most interesting way of expanding on these ideas is to introduce the games and activities which we developed with the children, using actual examples from these special sessions and examples which are of the same type as those which were used, though not the identical material. This will allow you to immerse yourself in the situation and give you a more complete idea of how and why the games fostered mathematical growth in the children and gave them pleasure in the process.

HOW TO USE THIS BOOK

The descriptions of games and activities in this book will differ somewhat from other teacher guides because they are intended to give both a partial account of our experiences in small group sessions and our suggestions to the teacher on how to proceed with an activity. We feel that the inclusion of our observations within an activity description will give the teacher a better idea of its potential for his or her small group sessions. Since our sessions were always with four children present, activities are described as if four students will participate. Most activities, however, may be used successfully with three, four or five students.

To insure and facilitate communication of the ideas and materials which were conceived or adapted for the small groups during the 1975 - 76 school year, it was, of course, necessary to establish an order of presentation for "Math Play Therapy". However, the order we have chosen for the book is in no way intended as an inflexible order to which a teacher must accede when using the material. In fact, we have not used that order ourselves in the tentative schedule at the end of this section, but have instead liberally mixed activities and games from all three chapters.

The tentative schedule is itself included simply to help anyone who might feel inexperienced and want additional help in structuring the year's work. We believe the individual teacher is the best judge of the children and the situation, and is, therefore, the person to make final decisions about what materials to use, and how and when to use them. He or she will be alert to the students' responses to specific activities and can adjust the materials or schedule accordingly. If you feel something is too difficult, not challenging enough or in some way ill-suited, we hope that you will not hesitate to adapt or eliminate it, whichever seems best.

Activities in Section 1.2, Let's Concentrate, are not included in the suggested schedule because they are warm-up exercises. We are suggesting only a main activity for each day and leave the selection of short warm-up activities to your discretion. Eventually, other activities in the same spirit as Let's Concentrate may be added, such as decoding of numbers on the Minicomputer to the hand-calculator display.

SUGGESTED SCHEDULE

We recommend that you schedule at least one supplementary session a week, and more when it is possible. For this reason we have included a 42-session schedule for small groups. If you are able to meet with your students only once a week, simply follow the schedule as far as time allows, saving the remaining sessions to be used the following year.

Session	Section Number	
1	1.1	Activity #1
2	3.1	Who Is Flip?
3	1.1	Activity #1
4	3.2	Who Is Kick?
5	1.1	Activity #2
6	3.3	Who Is Krack?
7	1.1	Activity #2
8	2.1	Minicomputer Tug-of-War
9	3.4	Who Is Zot?
10	2.1	Minicomputer Tug-of-War
11	1.1	Activity #3
12	3.5	Who Is Kwa?

Session	Section Number	
13	2.1	Minicomputer Tug-of-War
14	1.1	Activity #3
15	3.6	Who Is Kong?
16	2.1	Minicomputer Tug-of-War
17	3.7	Who Is Nim?
18	2.2	Minicomputer Golf
19	2.2	Minicomputer Golf
20	3.8	Who Is Tack?
21	2.1	Minicomputer Tug-of-War
22	2.2	Minicomputer Golf
23	3.9	Who Is Kim?
24	2.2	Minicomputer Golf
25	3.10	Who Is Pim?
26	2.2	Minicomputer Golf
27	3.11	Who Is Xan?
28	2.1	Minicomputer Tug-of-War
29	2.2	Minicomputer Golf
30	3.12	Who Is Gluck?
31	2.1	Minicomputer Tug-of-War
32	3.13	Who Is Tom?
33	2.2	Minicomputer Golf
34	3.14	Who Is Nock?
35	2.1	Minicomputer Tug-of-War
36	3.15	Who Is Jig?
37	2.2	Minicomputer Golf
38	3.16	Who Is Jag?
39	2.1	Minicomputer Tug-of-War
40	3.17	Who Is Jug?
41	2.2	Minicomputer Golf
42	3.18	Who Are Ko and Ku?

1

HAND-CALCULATOR

The activities described in this chapter use only the hand-calculator◊, unlike those in Chapters 2 and 3 which include other tools and languages. We had three goals in mind when developing and using these activities.

1) To give students a simple situation in which they can explore and attain some familiarity with the hand-calculator alone before being asked to use it in combination with other tools and languages.
2) To allow children time for free play in open situations where there are no wrong moves, in order to help them build confidence in themselves and in the hand-calculator.
3) To lay the groundwork for two Minicomputer games by using hand-calculator activities which are, in some sense, easier versions of those games.

Before any activity was initiated using the hand-calculator, the children were allowed to experiment with it on their own — not only on the first day, but usually for a few moments at the beginning of any session when it was used. Since we are in agreement with those educators who hold that play is an essential part of children's work and one of the primary ways in which they develop their understanding of new ideas and materials, we found this a very valuable first step in using hand-calculators. Given the nature of most children, it probably is a first step which they would have introduced, even if we had not!

◊For the following activities (and descriptions of them) to have any meaning, it is imperative that your hand-calculator have these two features: a) <u>algebraic logic</u> (response to instructions given in the order they are usually written): pressing $\boxed{2}$ $\boxed{+}$ $\boxed{2}$ and $\boxed{=}$ puts 4 on the display; and b) <u>constant mode</u>: pressing $\boxed{1}$ $\boxed{+}$ $\boxed{3}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ and $\boxed{=}$ puts 13 on the display.

1.1 Let's Play With Big Numbers

All of the following three activities involve large numbers because the students like the challenge of playing with them. Their self-concept is enhanced when they are given an opportunity to do so in a situation which is both open and simple enough that they can be successful (i.e., one which uses only addition and subtraction). The hand-calculators are essential for these games because they give the students a needed tool for calculating with large numbers.

The first game is obviously a very free situation which allows the children an opportunity to explore large numbers without the fear of making a "wrong move". It is our belief that some such free exploration is generally necessary in order for a child to make personal discoveries about numbers, and that such personal discoveries are the springboard to a deep fascination with numbers and the whole area of mathematical thinking.

In the second game there is still great freedom, but some limitations have been introduced to encourage the children to begin developing simple strategies.

Another expansion of skill is encouraged in the third activity by requiring the students to learn how to (approximately) place large numbers on the number line.

There is no strict period of time or order for these games. Any teacher choosing to use these activities will have to be sensitive to the children and to the situation in order to decide what game is most appropriate on a particular day, and when each game has outlived its usefulness.

Activity # 1:

Give each student a hand-calculator.

T: Let's start at 237. Everyone put 237 on the display. Now we will play together in this way: you take turns choosing for us either $\boxed{+}$ $\boxed{\text{some number}}$ or $\boxed{-}$ $\boxed{\text{some number}}$; we all push the chosen keys and $\boxed{=}$; and we examine the display. We are trying to reach 1,000 and the first student to make us hit our target is the winner.

Example:

		Display
First player:	+ 50	[287]
Second player:	+ 287	[574]
Third player:	+ 300	[874]
Fourth player:	+ 300	[1,174]

This example is from a game played in one of our small groups. At this point the first player hesitated for a long time. He concentrated deeply and mumbled, "I can't figure it out, I can't figure"; then slowly he announced "- 174", and was overjoyed when he read "1,000" or the display. This particular student is completely withdrawn in the classroom, is rarely successful in a collective lesson and is often laughed at by his classmates.

In our sessions, this game was played many times, with starting numbers generally between 100 and 400 and targets of 1,000, 2,000 or 3,000. The teacher never reacted during these games; she never praised or disapproved of any move. She deliberately chose to leave the students free to make their own experiments with the big numbers they hadn't yet mastered.

Activity #2:

T: Let's start with 123. Everyone put 123 on the display. The target will again be 1,000. You will take turns choosing $\boxed{+}$ $\boxed{\text{some number}}$ or $\boxed{-}$ $\boxed{\text{some number}}$, but this time you are working together to reach 1,000 in as few steps as possible.

Example:

		Display
First player:	+ 56	[179]
Second player:	+ 9,000	[9,179]
Third player:	- 300	[8,879]
Fourth player:	- 7,879	[1,000]

T: Your score is four. Play again and try to improve it. Put 123 on the display.

		Display
First player:	+ 789	[912]
Second player:	+ 83	[995]
Third player:	+ 6	[1,001]
Fourth player:	- 1	[1,000]

T: Your score is four again. Play once more. Put 123 on the display.

		Display
First player:	+ 600	[723]
Second player:	+ 23	[746]
Third player:	+ 385	[1,131]
Fourth player:	- 131	[1,000]

T: Your score is still four. Now each of you will play alone while the rest of us watch and record the results on our hand-calculators. Each time we start with 123 on the display.

		Display	
First player:	+ 456	[579]	
	+ 200	[779]	
	+ 389	[1,168]	
	- 168	[1,000]	Score: 4

		Display	
Second player:	+ 1,000	[1,123]	
	- 123	[1,000]	Score: 2

		Display	
Third player:	+ 900	[1,023]	
	- 23	[1,000]	Score: 2

		Display	
Fourth player:	+ 900	[1,023]	
	- 50	[973]	
	+ 50	[1,023]	
	- 23	[1,000]	Score: 4

The above example is an account of what occurred in one of our sessions. The fourth player was unable to give a better solution than with two steps, but he plays so as not to copy the third player. With another group of students, a similar activity (from 123 to 2,000) gave the following results.

		Display	
First player:	+ 1,000	[1,123]	
	+ 1,300	[2,423]	
	- 423	[2,000]	Score: 3

		Display	
Second player:	+ 30	[153]	
	+ 10	[163]	
	+ 40	[203]	
	+ 97	[300]	
	+ 100	[400]	
	+ 600	[1,000]	
	+ 1,000	[2,000]	Score: 7

A very methodical player!

		Display	
Third player:	+ 50	[173]	
	+ 173	[346]	
	+ 600	[946]	
	+ 1,000	[1,946]	
	+ 70	[2,016]	
	- 10	[2,006]	
	- 6	[2,000]	Score: 7

		Display	
Fourth player:	+ 923	[1,046]	
	+ 1,046	[2,092]	
	- 92	[2,000]	Score: 3

The first player asked to play again and reached the target in two steps:

		Display
First player:	+ 2,000	[2,123]
	- 123	[2,000]

One week later, we worked on a similar problem (456 to 1,000). In one group where the players had previously solved the problems with scores varying from five to nine, one student claimed that he could reach the target in two steps and he played:

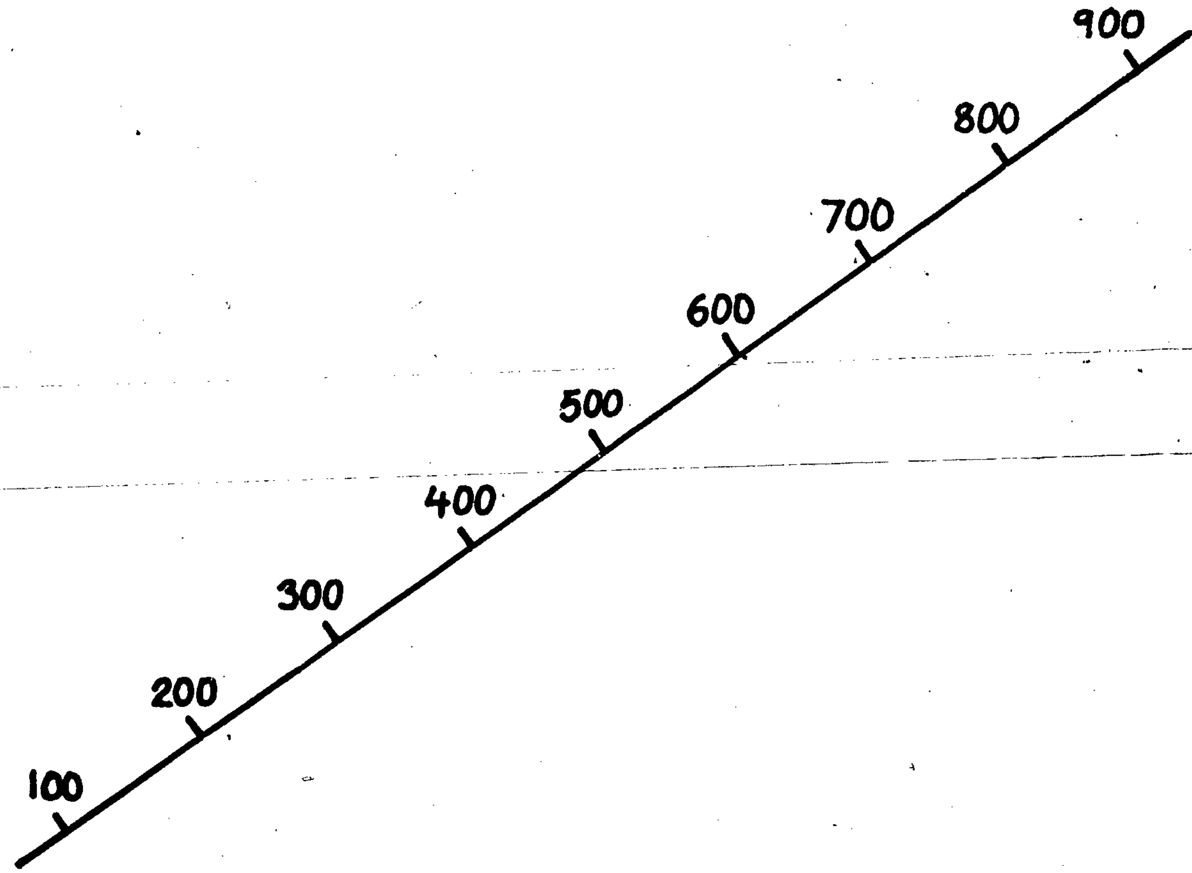
	Display
- 456	[0]
+ 1,000	[1,000]

Immediately afterwards, the same student announced that he could solve the problem in one step. After two tries he succeeded [+544].

The free play in Activity #1 laid the groundwork for Activity #2 which is more structured and problem-solving oriented. The challenge and the joy of working with big numbers stimulated the students to the point that some of them found ingenious strategies for solving a problem in two steps. We believe that these activities gave some of the students their first chance to make a real discovery in mathematics. It was a turning point in their perception of what is an intellectual activity and in their desire to be involved in subsequent lessons.

Activity #3:

Divide the group of students into two teams (A and B) and draw this number line on the board.

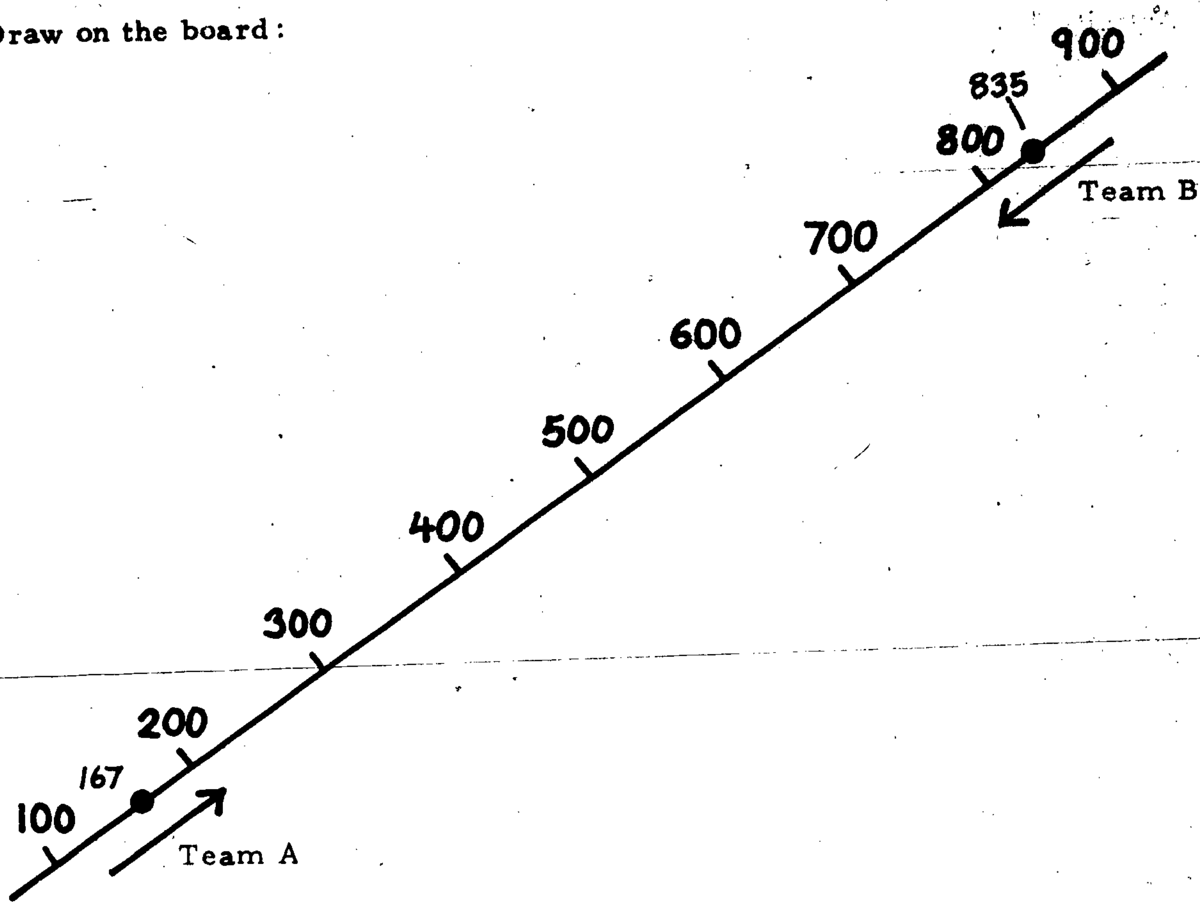


T: The students on team A will start from 167 and the students on team B from 835.

A student from each team has to point to the approximate location of his or her number on the number line.

T: The students on team A can choose only $\boxed{+}$ $\boxed{\text{some number}}$, and the students on team B $\boxed{-}$ $\boxed{\text{some number}}$. Again you take turns. The first team to meet the other team or to pass it will be the LOSING team.

Draw on the board:



Each time a student plays, he or she has to mark the approximate location of the team's new number on the number line. (It would be helpful to have the teams use different colors to locate their numbers, say Team A: red and Team B: blue.)

Example of a game:

		Display
Team A:	+ 67	[234]
Team B:	- 40	[795]
Team A:	+ 70	[304]
Team B:	- 200	[595]
Team A:	+ 20	[324]
Team B:	- 40	[555]

		Display
Team A:	+ 100	[424]
Team B:	- 55	[500]
Team A:	+ 60	[484]
Team B:	- 10	[490]
Team A:	+ 5	[489]

Since these students do not, at this point, have a working knowledge of the non-integer decimal numbers, they most likely will conclude that team B has lost.

This game should be played several times.

1.2 Let's Concentrate

The following activities involving the hand-calculator are in the same form as those which were used with the small groups during our experimentation, although they may not be precise examples of the actual games played. They are presented here not as "the way to proceed", but rather in the spirit of sharing an idea which seems rich in possibilities; one which we hope will encourage experimentation on the part of individual teachers and faster growth for their particular students.

While writing the activities, these short-term goals were kept in mind:

- To help the students develop the ability to concentrate with some intensity for a short period of time.
- To familiarize the students with the use of the hand-calculator in executing the four basic operations.
- To present an open activity which encourages a child's

creativity in using numbers by allowing him or her to choose any one of many possible ways to get from one number to another.

It should be stressed that there is no rigid schedule for the use of these exercises. The teacher is urged to be completely free in using them. We believe that the more variety a teacher introduces into these activities, the better the results will be. If one seems inappropriate or fails to generate a response in the students that day, you may either switch to a different choice of numbers and operations or discontinue the activity for that session. In all cases, it is recommended that these activities be used as short warm-up exercises for no more than five to ten minutes at the beginning of a session.

For all of these activities, each student is given a hand-calculator. The teacher gives the instructions slowly and distinctly.

Activity #1:

T: We will start from 0. Check that 0 is on the display.

Press $\boxed{2}$ $\boxed{\times}$ $\boxed{5}$ $\boxed{+}$ $\boxed{3}$

Hide the display (with a small piece of paper or with your hand).

Press $\boxed{=}$

What number should be on the display? [Answer: 13]

Look to see if it's correct. You may continue to look at the display for the moment.

Press $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$.

What number is on the display? [Answer: 25]

What do you notice? [Answer: it's counting by threes.]

Again press $\boxed{=}$ $\boxed{=}$ $\boxed{=}$.

What number is on the display? [Answer: 34]

Hide the display again.

Press $\boxed{=}$ $\boxed{=}$.

What number should be on the display? [Answer: 40]

Look to see if it's correct, then hide the display again.

Press $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$.

What number should be on the display? [Answer: 52]

Look to see if it's correct. You may look at the display this time.

Press $\boxed{+}$ $\boxed{1}\boxed{0}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$.

What number is on the display? [Answer: 82]

What do you notice? [Answer: it's counting by tens]

Hide the display again.

Press $\boxed{=}$ $\boxed{=}$.

What number should be on the display? [Answer: 102]

Look to see if it's correct and then hide the display again.

Press $\boxed{=}$ $\boxed{=}$ $\boxed{=}$.

What number should be on the display? [Answer: 132]

Look to see if it's correct. You may look at the display now. Do anything you wish, but try to end with 100 on the display.

After a while, the teacher asks the students to explain how they went from 132 to 100 and encourages them to give several answers. For instance,

$\boxed{-}$ $\boxed{3}\boxed{2}$ $\boxed{=}$

$\boxed{-}$ $\boxed{2}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ . . . until they get 100.

$\boxed{-}$ $\boxed{1}\boxed{0}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ $\boxed{-}$ $\boxed{2}$ $\boxed{=}$

$\boxed{-}$ $\boxed{1}\boxed{3}\boxed{2}$ $\boxed{+}$ $\boxed{1}\boxed{0}\boxed{0}$ $\boxed{=}$

and so on.

Activity #2:

T: Check that 0 is on the display, then hide the display.

Press $\boxed{3}\ \boxed{\times}\ \boxed{5}\ \boxed{+}\ \boxed{4}\ \boxed{=}$.

What number should be on the display? [Answer: 19]

Look to see if it's correct. You may look at the display this time.

Press $\boxed{=}\ \boxed{=}\ \boxed{=}$.

What number is on the display? [Answer: 31]

What do you notice? [Answer: it's counting by fours]

Hide the display again.

Press $\boxed{=}\ \boxed{=}$.

What number should be on the display? [Answer: 39]

Look to see if it's correct, then hide the display again.

Press $\boxed{=}\ \boxed{=}\ \boxed{=}\ \boxed{=}$.

What number should be on the display? [Answer: 55]

Look to see if it's correct. Hide the display. Do anything you wish, but try to end with 100.

If some students do not succeed, the teacher asks them to do the same exercise by looking at the display. She encourages them to solve this problem in different ways and to explain their solutions.

Activity #3:

T: We start from 1,000; put 1,000 on the display. Then hide the display.

Press $\boxed{-}\ \boxed{2}\boxed{0}\boxed{0}\ \boxed{=}$.

What number should be on the display? [Answer: 800]

Look to see if it's correct and then hide the display again.

Press $\boxed{\div}\ \boxed{2}\ \boxed{=}$.

What number should be on the display? [Answer: 400]

Look to see if it's correct. You may look at the display this time.

Press $\boxed{=}$ $\boxed{=}$.

What number is on the display? [Answer: 100]

Hide the display again.

Press $\boxed{=}$.

What number should be on the display? [Answer: 50]

You may look at the display this time.

Press $\boxed{-}$ $\boxed{3}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$.

What number is on the display? [Answer: 38]

Hide the display again.

Press $\boxed{=}$ $\boxed{=}$ $\boxed{=}$.

What number should be on the display? [Answer: 29]

Look to see if it's correct and then hide the display again.

Press $\boxed{=}$ as many times as necessary to end with 20.

Look to see if it's correct. How many times did you press $\boxed{=}$?

[Answer: three times]

Hide the display again.

Press $\boxed{=}$ any number of times, until you think the display will show a negative number for the first time.

What is this negative number? [Answer: $^{-}1$]

Look to see if it's correct. You may look at the display this time.

Press $\boxed{=}$ $\boxed{=}$ $\boxed{=}$.

What number is on the display? [Answer: $^{-}10$]

Hide the display again.

Press $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$.

What number should be on the display? [Answer: $^{-}22$]

Don't hide the display any more. Do anything you wish but try to end with 10.

After a while the teacher asks the students to explain how they solved this problem and encourages them to try to find several solutions.

Activity #4:

T: We start from 0.

Press $\boxed{+}\ \boxed{4}\ \boxed{=}\ \boxed{=}\ \boxed{=}\ \boxed{=}$.

What number is on the display?

S: 16.

T: Hide the display.

Press $\boxed{=}\ \boxed{=}\ \boxed{=}$.

What number should be on the display now?

S: 28.

T: Check it. Hide the display again.

Press $\boxed{=}\ \boxed{=}\ \boxed{=}$.

What number should be on the display now?

S: 40.

T: Check it. Hide the display again.

Press $\boxed{=}$ as many times as necessary to get 60 on the display.

T: Check that you have 60.

If we continue to press $\boxed{=}$, what is the first number that will appear on the display ending with "00"?

S: 100.

T: Check that you can get 100.

Press $\boxed{-}$ $\boxed{3}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$.

What number is on the display?

S: 88.

T: Hide the display.

Press $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$.

What number should be on the display?

S: 76.

T: Check it. Hide the display again.

Press $\boxed{=}$ as many times as necessary to get 61 on the display.

T: Check that you have 61 on the display. Hide the display.

By continuing to press $\boxed{=}$, can you get 40 on the display?

Probably many students will answer "yes".

T: Check that you can get 40 this way.

Now you have 40 on the display.

Hide the display again and continue to press $\boxed{=}$ until you think the smallest positive number is on the display.

T: Now look and tell me the number.

S: 1.

T: Hide the display again. Do any calculation you wish to get 20.

Many answers are possible. For instance,

$\boxed{+}$ $\boxed{9}$ $\boxed{+}$ $\boxed{1}\boxed{0}$

$\boxed{+}$ $\boxed{1}\boxed{9}$

$\boxed{+}$ $\boxed{9}$ $\boxed{\times}$ $\boxed{2}$

$\boxed{-}$ $\boxed{1}$ $\boxed{+}$ $\boxed{2}\boxed{0}$

$\boxed{\times}$ $\boxed{2}\boxed{0}$

and so on.

T: Check to see that you really have 20 on the display.

Hide the display again.

Do a calculation to get 100.

Many answers are possible. The teacher encourages the students to find different possibilities. For instance,

$\boxed{+}$ $\boxed{8}\boxed{0}$

$\boxed{\times}$ $\boxed{5}$

$\boxed{+}$ $\boxed{1}\boxed{0}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$

$\boxed{-}$ $\boxed{2}\boxed{0}$ $\boxed{+}$ $\boxed{1}\boxed{0}\boxed{0}$

$\boxed{+}$ $\boxed{5}$ $\boxed{\times}$ $\boxed{4}$

She also encourages the students to give solutions using at least two operational signs.

2

MINICOMPUTER GAMES

The two main implements which we have used in our work with the small groups are the hand-calculator (used in activities described in Chapters 1, 2 and 3) and the Minicomputer◇ (used in activities presented in Chapters 2 and 3). There are some important observations which should be recorded here about the complementary symbiotic relationship which we believe exists between these two instruments.

When we used them together with slow learners, we could see that each strengthens and enriches the therapeutic value of the other. Used together, they constitute a very strong set of tools from which the students can create and build a solid structure of mathematical knowledge.

Because the hand-calculator has buttons to manipulate and a display to watch, it helps children to focus on their own individual efforts. Because the children are so intent, they are less likely to give up in their effort to solve a particular problem or, if they have solved it, to call out the answer, spoiling it for others who are still working.

The Minicomputer, on the other hand, takes them in just the opposite direction; it allows them physical and mental movement, shows them new ways of writing the same number and, in general enlarges, rather than focuses the scope of their understanding about a particular number.

It is a very important moment when a child can look at a number on the Minicomputer, then press the correct buttons on the hand-calculator and make this number appear on the display. It is at that moment, we believe,

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◇Readers unfamiliar with the Minicomputer are referred to the Appendix on page 58.

that the child realizes the identity of the number, and that this number may be expressed in many ways but always retain its identity.

Since the complementary relationship between the hand-calculator and the Minicomputer stimulates the student's mathematical development so successfully, we used the tools simultaneously for almost all the activities with the small groups.

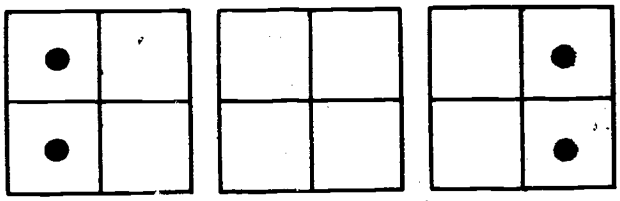
It was during the academic year, while we worked regularly with the students and became increasingly concerned with finding ways to break through the barrier of their past experiences of failure, that the ideas were conceived for two different games which could be played on the Minicomputer. As we worked with these games, we became more and more aware of their capacity for activating the numerical thinking of the children and for promoting strategical thinking. It is the goal of this chapter to describe these games and to define the potentially therapeutic role we believe they can play in any situation where children are discouraged and performing poorly.

2.1 Minicomputer Tug-of-War

In this section we describe the Minicomputer Tug-of-War activity, a game which seems to have exciting potential for work with slow learners. Since this game was invented at the end of the year, we did not have the opportunity of using it in a systematic way in our small groups. However, we were able to arrange some special sessions during summer school. In these sessions we used a group of children who had just completed the third grade part of the CSMP curriculum and were enrolled in summer school on a voluntary basis. We found the game very beneficial and suitable for children at this level. We recommend that it be used at the beginning of fourth grade as a natural preparation for the more complex game of Minicomputer Golf.

We have thought carefully about the characteristics of this game which make it so appropriate and helpful, but before we report our thinking about that aspect, we will describe the game using two examples from games the children played. The first example is the fifth game that they played. All of the preceeding games used two checkers per team.

The teacher displays three Minicomputer boards and puts on two yellow and two blue checkers in this way:



She gives a hand-calculator[◇] to each student and asks them to choose partners.

T: Team A will play with the yellow checkers and Team B with the blue checkers. What is the "yellow number"? (5)

She asks the students of Team A to display 5 on their hand-calculators.

T: What is the "blue number"? (1,000)

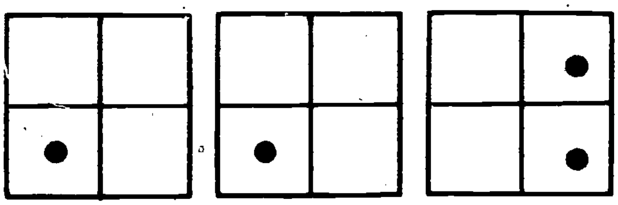
◇Although we used the hand-calculator here to stimulate concentration, we leave its use as an option. It is not intrinsically necessary to the students' successful participation in the game, particularly when only two checkers are used.

She asks the students of Team B to display 1,000 on their hand-calculator and writes on the board:

1,000	5

T: Students from Team A and from Team B will take turns playing. Players of Team A have to move one of their checkers to a square that has a <u>higher</u> value. Players of Team B have to move one of their checkers to a square that has a <u>lower</u> value. So the "yellow number" will increase while the "blue number" will decrease. The first team that ties or passes the other ("yellow" becomes equal to or greater than "blue"; "blue" becomes equal to or less than "yellow"), LOSES the game.

A student from Team B plays first and moves one blue checker this way:

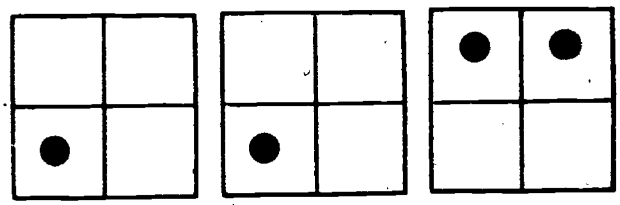


T: What is the "blue number" now? (220)

The students of Team B display 220 on their hand-calculators. The teacher erases "1,000" from the blue frame and writes "220" in it.

220	5

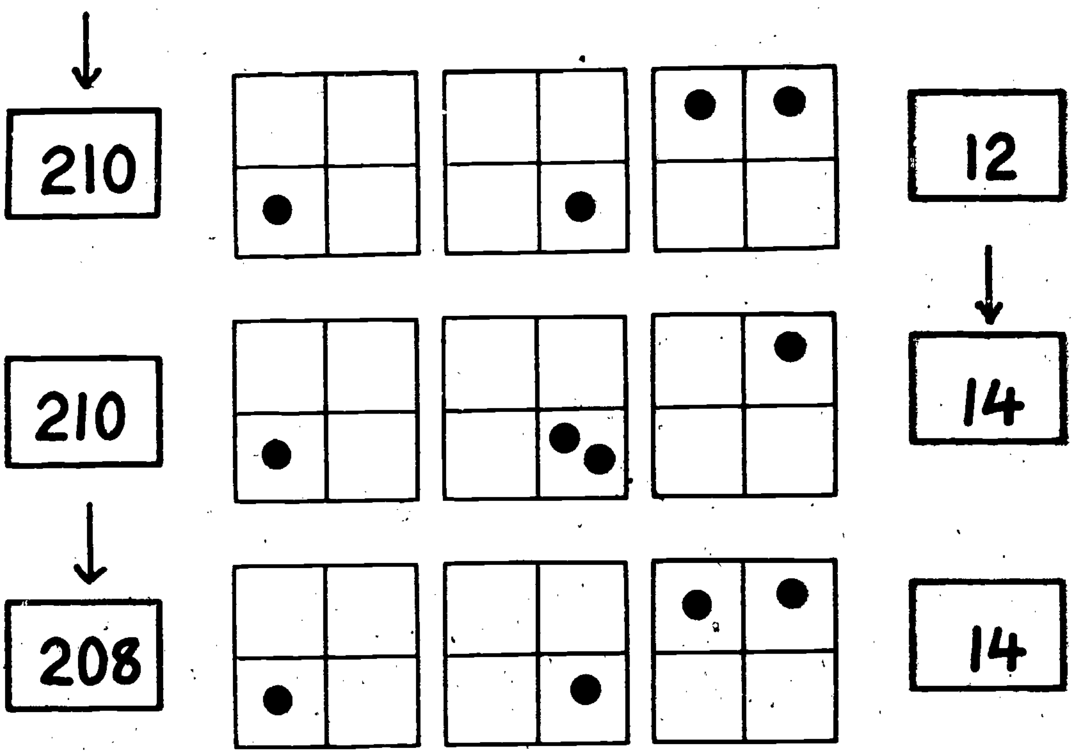
A student from Team A moves one yellow checker this way:

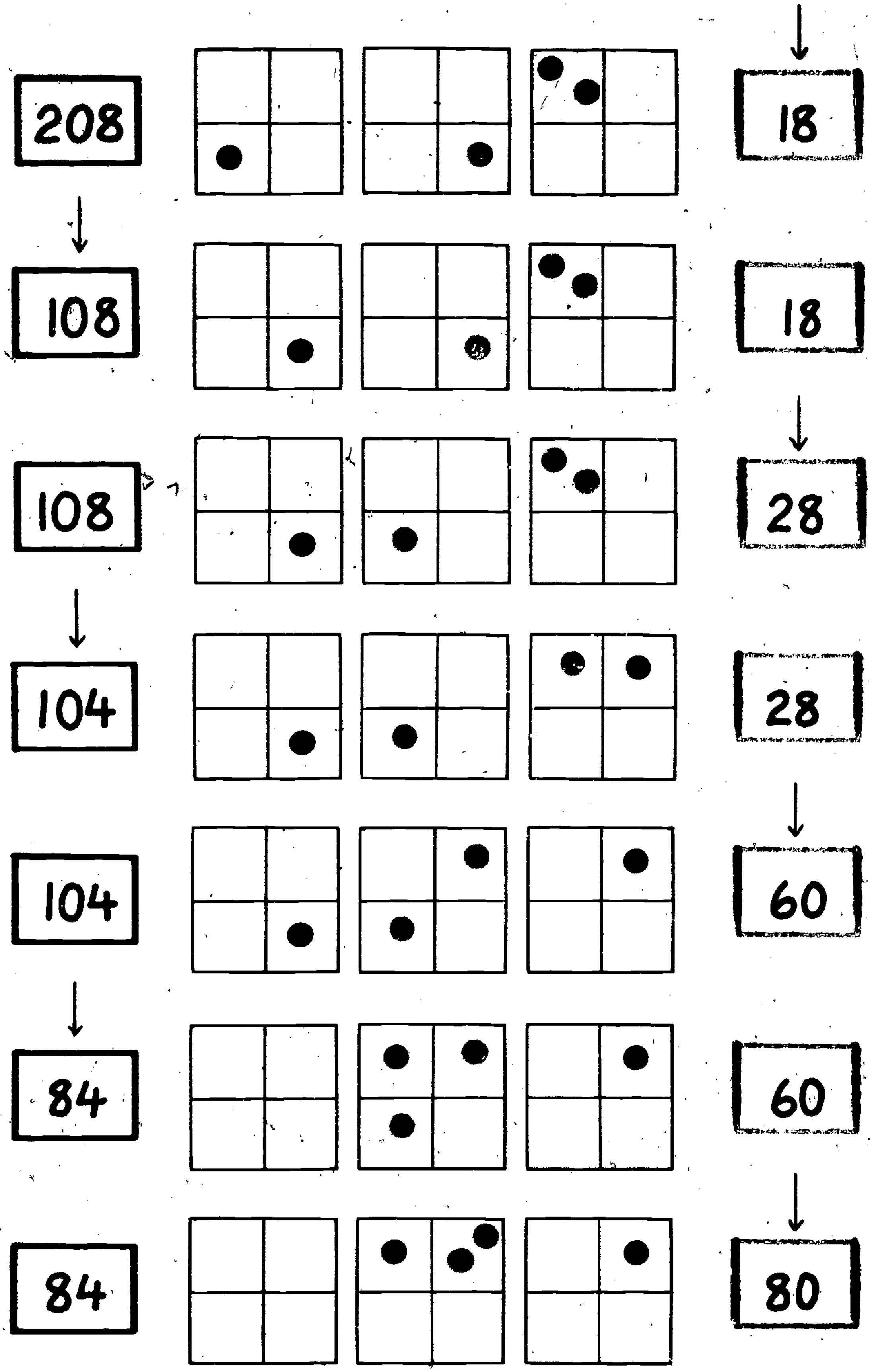


T: What is the "yellow number" now? (12)

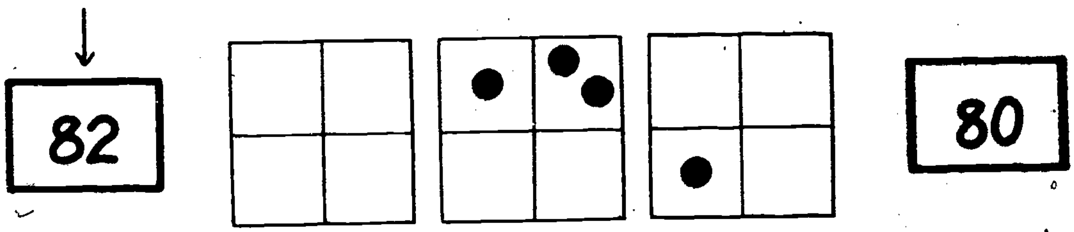
The students of Team A display 12 on their hand-calculators. The teacher erases "5" from the yellow frame and writes "12" in it.

The next steps are described below:




208
18
108
18
108
28
104
28
104
60
84
60
84
80

At this point, the student whose turn it is smiles happily and exclaims, "I can make them lose!" She moves in this way:

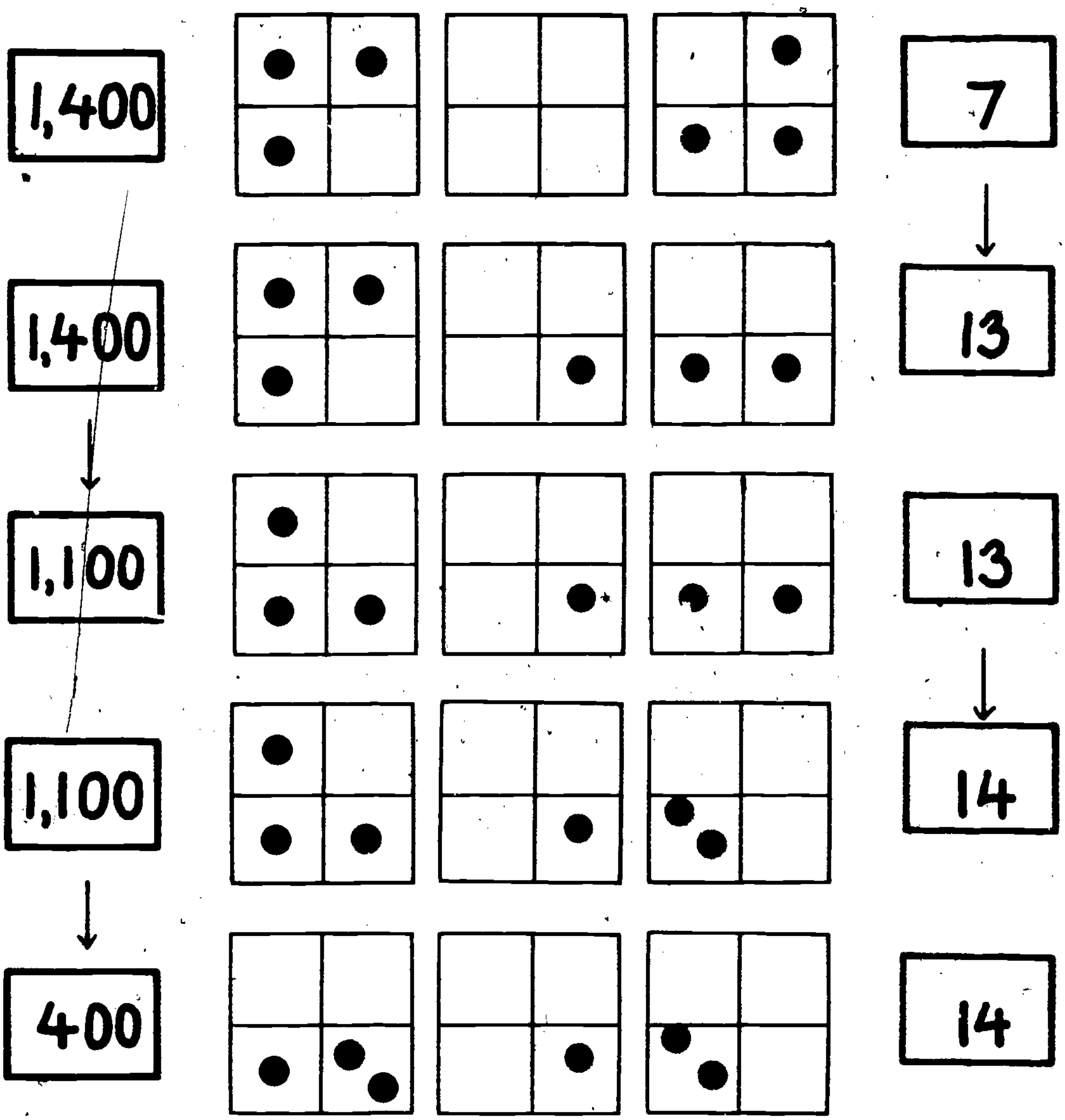


Team A surveys the situation and quickly concedes that she is right, because their smallest possible move is from the 40-square to the 80-square. Since their new number would be 120, they are certainly the losers.

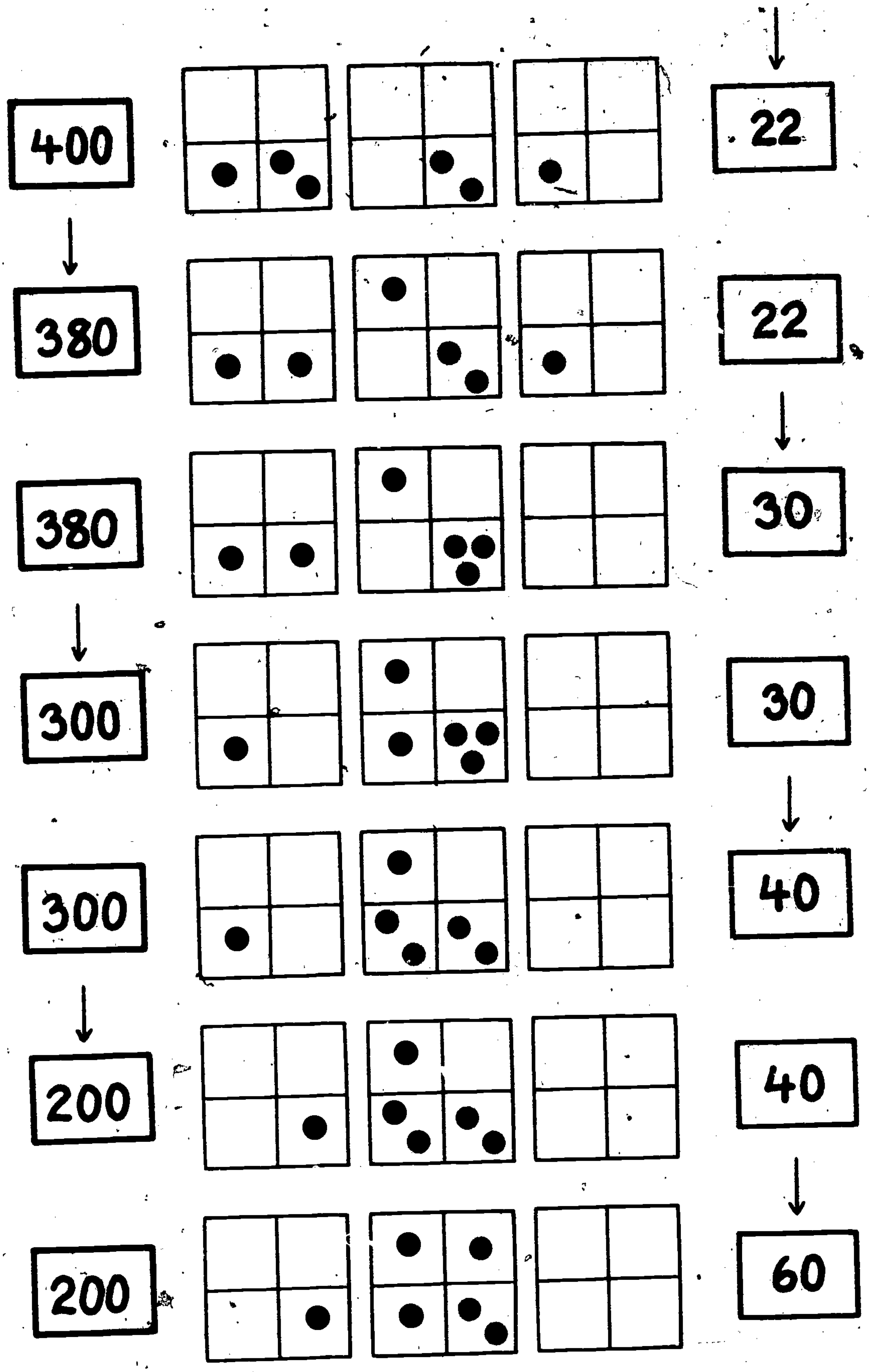
If you make a careful study of the preceeding game, you will probably observe that after "yellow's" second move (to 14, when "blue" was at 210), it would have been possible for "blue" to win by moving from the 200-square to the 8-square. Thus, "blue" would have been 18 and "yellow" (at 14) could only meet or pass 18, thereby becoming the loser. However, the children were not playing on this sophisticated level, nor were they expected to be. This sort of situation occured several times during the game but the teacher simply noted it without comment. Instead, she allowed the children to have an enjoyable time making their rather cautious moves toward each other. They were easily able to determine each new number and record their own team's number on the hand-calculator. But it was only when the two numbers were quite close that one of the students was able to see and verbalize what was happening, and find a winning move.

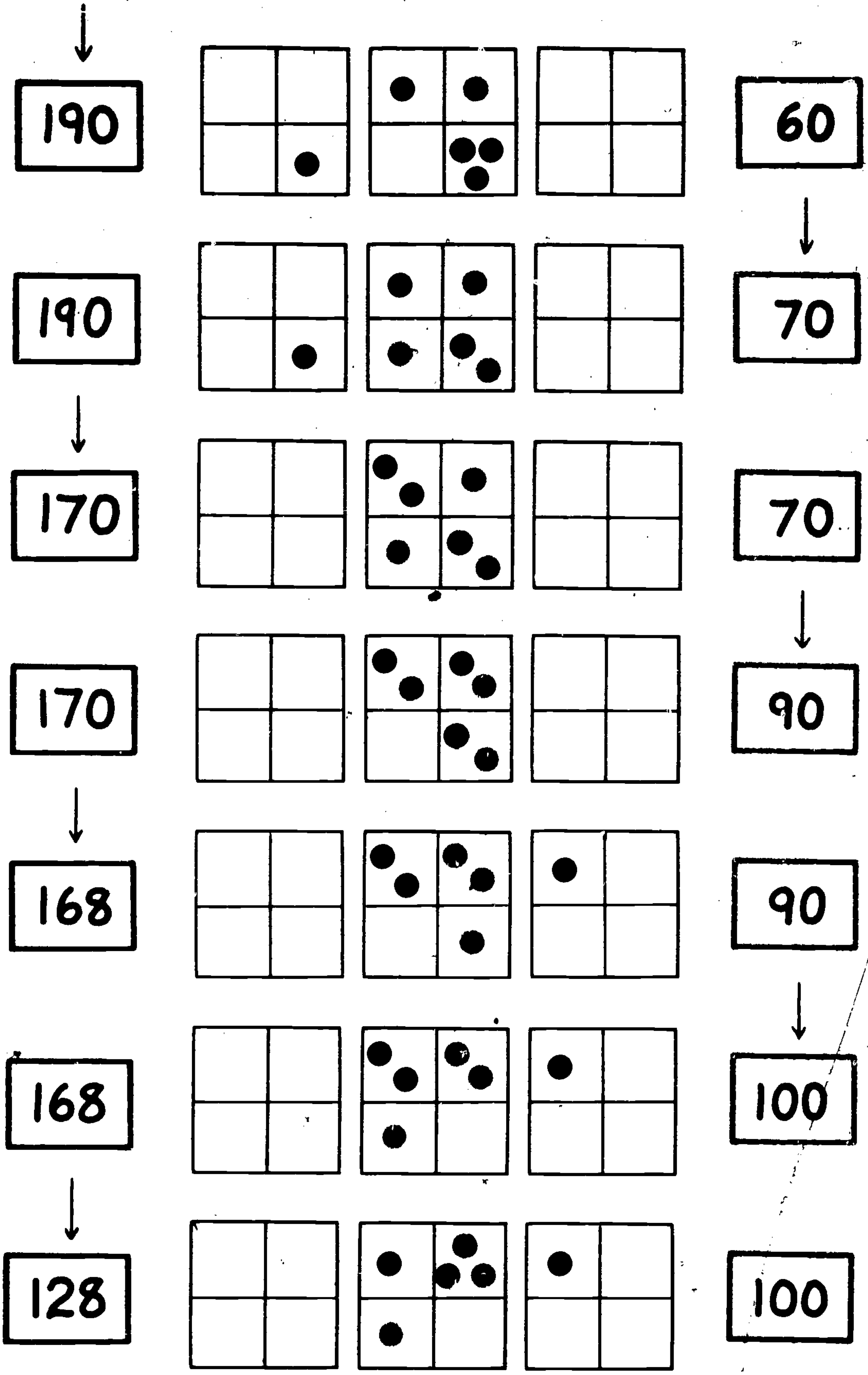
It is easy to see that this game, when played on the simplest level, is well within the range of abilities of even the lowest achieving students. If they can decode numbers which use only two checkers and if they can compare two such numbers to determine which is bigger or smaller, they can play and enjoy this game. As they are playing, they are improving their ability to decode numbers (at first with only two checkers, later with three and eventually four checkers). They are also increasing their ability to estimate the distance between two numbers. As their sophistication increases, they begin to anticipate what their opponent's next move may be and in this way they begin naturally to discover strategies which will help them win.

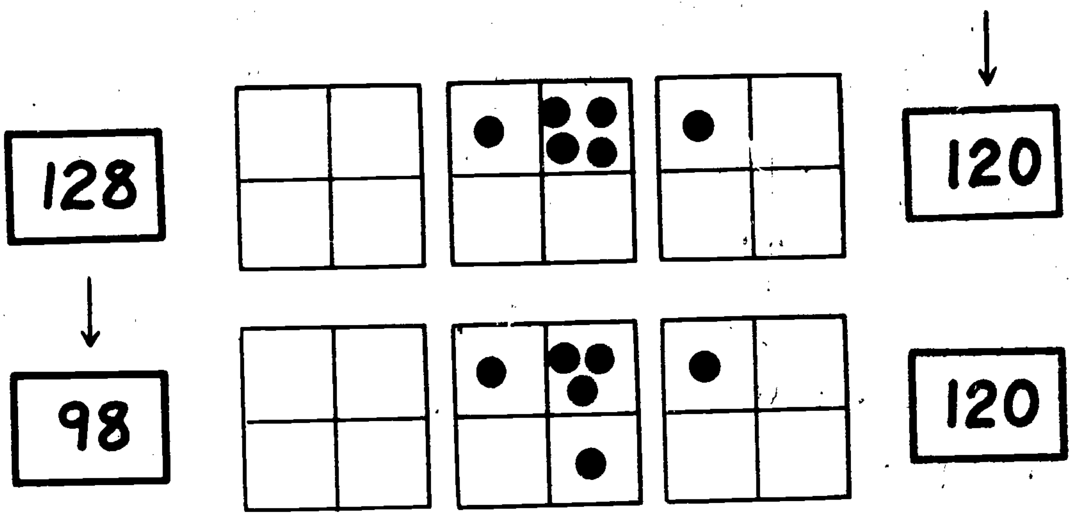
We now include an example[◇] of the game played with three checkers per team. This is the next natural step. The rules are identical.



[◇]This was the sixth game the children played and the first one in which they used three checkers.


400
22
380
22
380
30
300
30
300
40
200
40
200
60


190
60
190
70
170
70
170
90
168
90
168
100
128
100



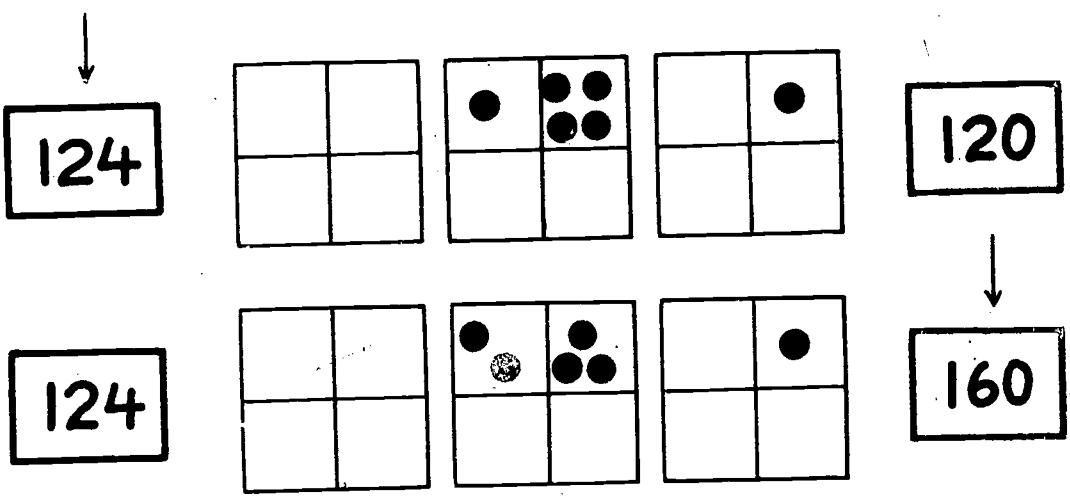
T: What do you think has happened?

S (from "blue" team): We lost!

T: Yes, that's right. Let's put the last checker back where it was before and see whether you had to lose. (The teacher shows the boards as they were at $\boxed{128}$ and $\boxed{120}$.) What do you think?

S (from "blue" team): We could have moved a checker from the 8-square to the 4-square and then they would have lost!

The student demonstrates this fact as shown below:



In this more complex game, the children were less able to anticipate where their moves were taking them and to use a winning strategy. Actually, none of them were aware that "blue" had lost for a moment. No matter! The teacher simply asked a question to direct their thoughts to what had taken place, and some of them understood. A further question helped one child project another possible outcome.

Because we were able to schedule only a few sessions with the children in summer school, we had to proceed more rapidly from the two-checker to the three-checker game than we would have under normal circumstances. We would suggest a gradual progression from two-checker to three-checker games in the earlier part of the year, interspersing them with other activities. This will give the children time to improve their ability to decode numbers and estimate distances, and also time to begin anticipating moves which the opposing team might make and to develop strategies which are more advantageous to their team.

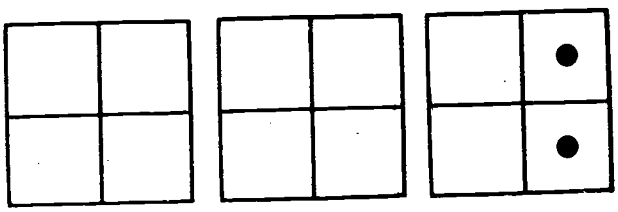
In order to give them the widest variety of experiences, the teacher should shift teams (or team members) from the "blue" to the "yellow" side and vice versa. This will encourage the students to create new ways of playing the game and familiarize them with "blue" and "yellow" strategies, which are different.

Later in the year, after they have also had experience with Minicomputer Golf, the teacher should return again to this game in order to give the students the exposure and encouragement that will help them find strategies of increasing sophistication. Perhaps at that time he or she will want to introduce four checkers ("blue" starting at 1,500 and "yellow" at 15).

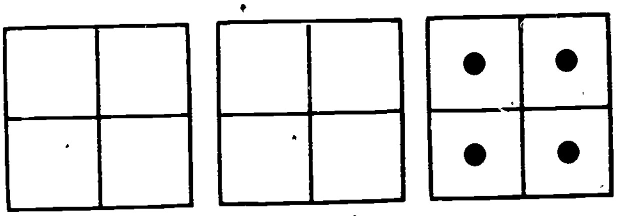
2.2 Minicomputer Golf

In this section we will present Minicomputer Golf, a game which has had a compelling attraction for students, teacher and observer alike in the work of the small groups. We now understand from our experience and careful study of that experience, that this game has played a key role in the success of these groups. We have given a generous amount of time and thought to the question of why it has had such a positive effect and will later explore many of the reasons that seem to have a bearing on this question. However, it is first important that we explain the game and the way we used it. In order to do this, we will reproduce one of the many games which was actually played and make a few observations about it.

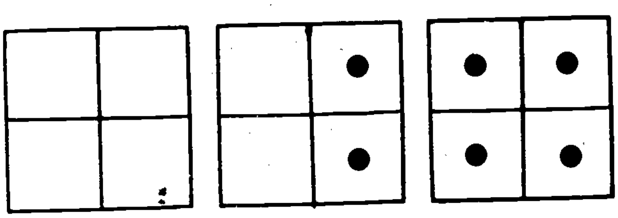
Eight checkers are gradually set up on the Minicomputer, beginning with the ones' board. After each new addition of checkers, a child is called on to read the number. For example,



S_1: 5

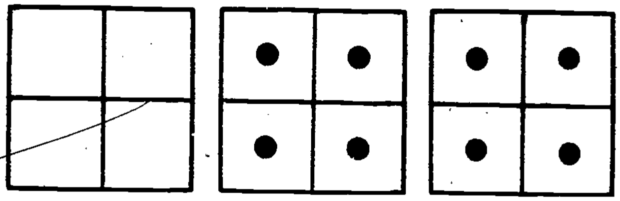


S_2: 15



S_3: 65

Finally, the last two checkers are put on so that the starting situation using three boards is:



S_4: 165

The students have already received hand-calculators and the teacher now instructs them to put 165 on the calculators' displays. After the children have chosen partners, the teacher proceeds as we will show below. We have used as an example a game which was played after the children had already had three other experiences with the game.

T: We are starting from 165 and we have to reach 400. Students from Team A and from Team B will play in turn. Each time, you may move one checker from any square to any other square. The number may be increased or decreased. The first team to reach the goal will be the winner. Who would like to play first for Team A?

The first player on Team A moves a checker from the 20 - square to the 80 - square.

T: Is the number on the Minicomputer bigger or smaller? (Answer: bigger)

How much bigger? (Answer: 60 bigger)

What number is on the Minicomputer now? (Answer: 165 + 60)

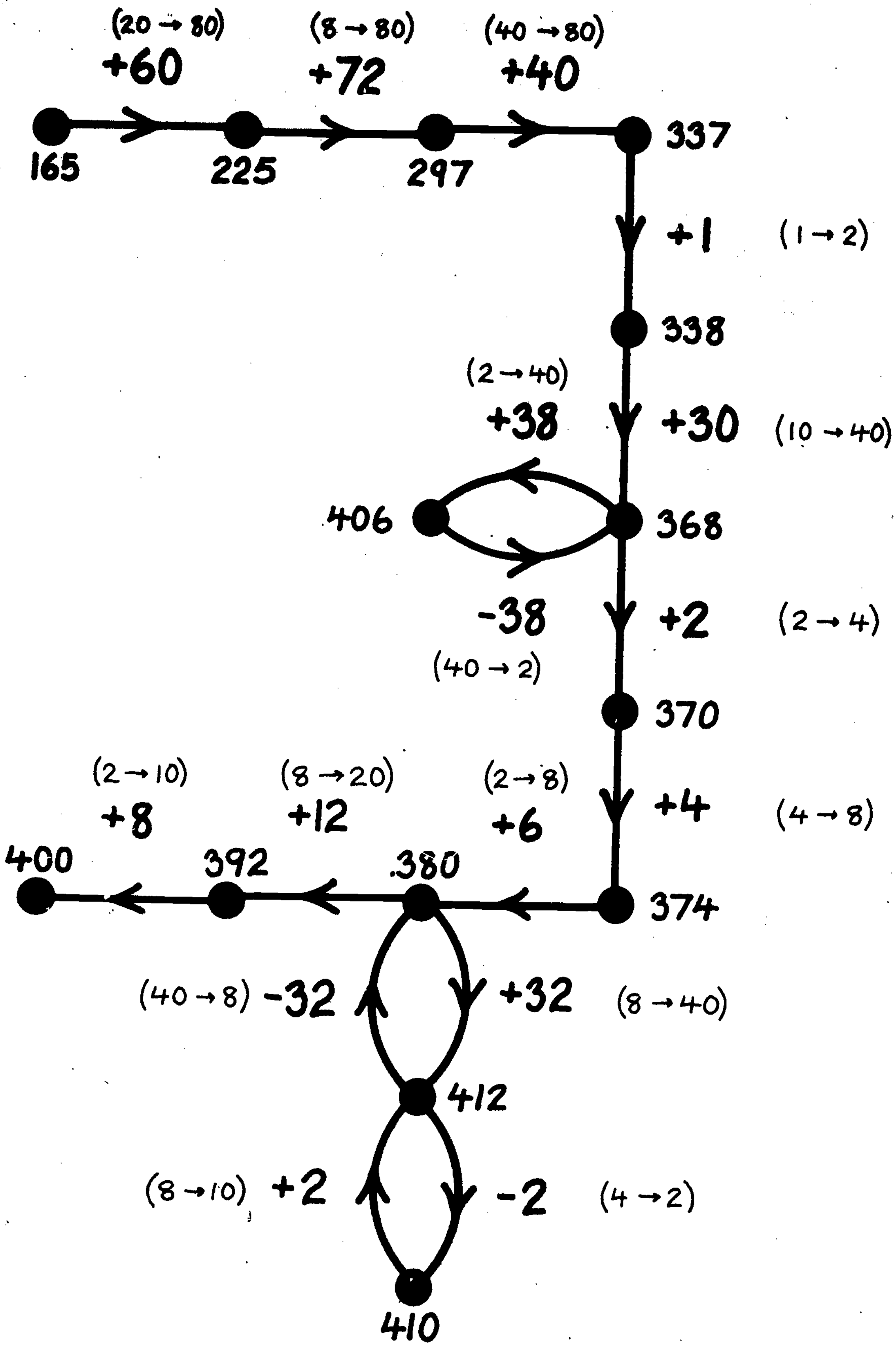
The students record the new number by pressing $\boxed{+}$ $\boxed{6}$ $\boxed{0}$ $\boxed{=}$. Next a player from Team B moves a checker from the 8 - square to the 80 - square.

T: Is the number on the Minicomputer bigger or smaller? (Answer: bigger)

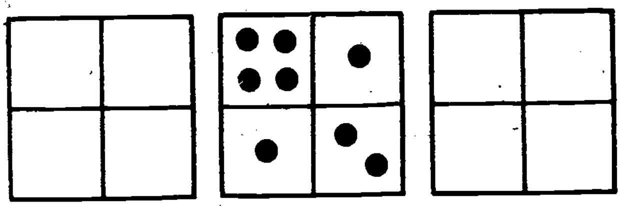
How much bigger? (Answer: 72 bigger)

What is the new number on the Minicomputer? (Answer: 225 + 72)

The students again record the new number. Similar steps are followed with the next players, alternating from Team A and Team B. If the player who makes the move does not know how much the number has increased or decreased, one of the other students is allowed to give the answer. If none of them knows, the teacher helps them figure it out or asks them to choose an easier move. The results of the game are described by this red - blue road. Team B (blue arrows) was the winner.


(20 → 80)
+60
(8 → 80)
+72
(40 → 80)
+40
165
225
297
337
+1
(1 → 2)
338
(2 → 40)
+38
+30
(10 → 40)
406
368
-38
(40 → 2)
+2
(2 → 4)
370
(2 → 10)
+8
(8 → 20)
+12
(2 → 8)
+6
+4
(4 → 8)
400
392
380
374
(40 → 8) -32
+32
(8 → 40)
412
(8 → 10) +2
-2
(4 → 2)
410

In the arrow picture on the preceeding page, as well as in all of those which follow, the notation "x ⟶ y" indicates a move from the x - square to the y - square. The final boards had this configuration:



= 400

Contrary to our practice with the collective lessons, we did not record the individual moves on the board by drawing an arrow picture for the small groups (although the observer made a record of them). This was a deliberate choice and we would like to emphasize that it was a personal one, based on a definite goal which was, for us, more important than any other consideration.

Our reason for omitting the record-keeping was that our major goal was to focus deeply on the students and on our relationship with them. The mechanics of keeping a record would have been both distracting and a barrier between us.

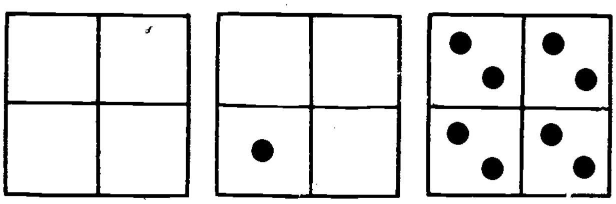
However, we are aware of the value of the arrow picture in that it points to and reinforces the basic connection between the language of arrows and the Minicomputer language. It may be that for another teacher this will be a more important consideration, and that the choice will be made to keep the record in the same way it is kept in a collective lesson.

Now, let us look at some of the more important observations we have made about this game and the students' response to it; in particular, to those observations which seem to embody a reason for its very therapeutic effect upon the children.

1) The game may be played many, many times without being exactly the same, even if the starting and ending numbers are the same. This is because there are so many different roads which are possible, depending on the way the checkers are set up and the combination of moves that are chosen by the children.

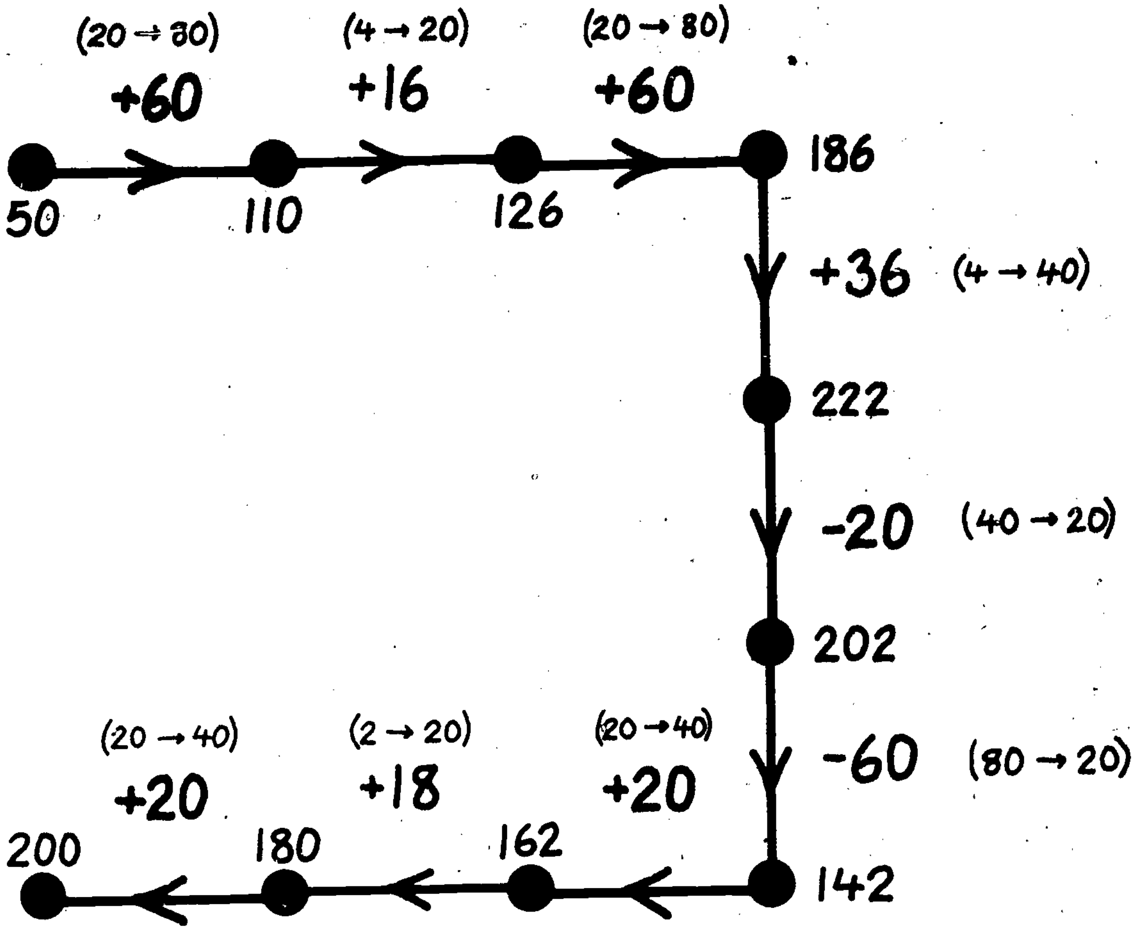
In addition, the starting and ending numbers chosen may be small (however, at least seven checkers should be used) or large. This gives the game great flexibility and puts it easily within the range of slow learners, even on their very first try. For example, here is a situation we set up as one of the first experiences with Minicomputer Golf.

Starting boards:

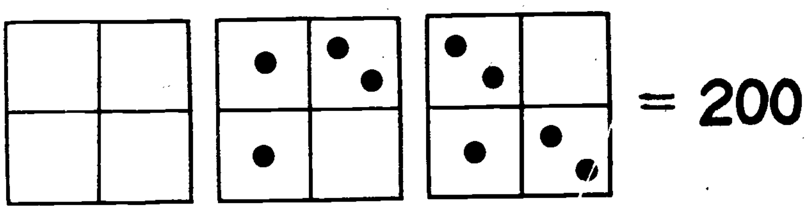


= 50

Goal: 200



The final boards appeared as below:



The children enjoy this game and are able to get to the goal without too much difficulty, although it is clear their strategy is more haphazard than purposeful.

2) Even if students are very weak, they can play this game because there are no wrong moves. Because it is always possible to get to the goal, the weaknesses of individual children do not spoil the game for the others.

A clear illustration of this is shown on page 42 in moves 11 - 14. Although the children take many roundabout paths during these moves, they do eventually reach their goal — and have fun in the process!

The teacher, here and always, accepts any move in silence, without any comment or comparison. She is also careful that her expression does not communicate some positive or negative judgment. Because she allows all to play as they like and are able, and because she has the attitude of "enjoy the game and see what happens," the students are truly free. They are not afraid to explore, and they quickly understand that to go above the goal may be a very sensible move in their journey toward it.

3) As the game is being enjoyed for itself, each student is also expanding his or her knowledge of some basic facts and developing the ability to estimate distance more accurately at his or her own rate of speed. Each is making individual discoveries which are not spoiled even if they occur long after they have been made by others, because they are a personal happening.

We reiterate that we deliberately chose to emphasize the enjoyment of the game and to remove, as far as possible, those elements of comparison, judgment and competition which are always present in a group. Nevertheless, we will admit to some surprise at what a natural teacher the game is, when used in this way. Because a child can play and enjoy the game even with a very poor sense of the distance between two given numbers, that child slowly and naturally develops within himself or herself the ability to estimate distances more accurately. In the same way, when students are not put under constant pressure to come up with basic facts about numbers, but are instead immersed in a situation which repeatedly exposes them to those facts, they simply absorb some of them as a matter of course. If, as we did, the teacher stimulates thinking about estimation with questions which are asked in a casual, non-systematic way, the learning appears to be increased significantly.

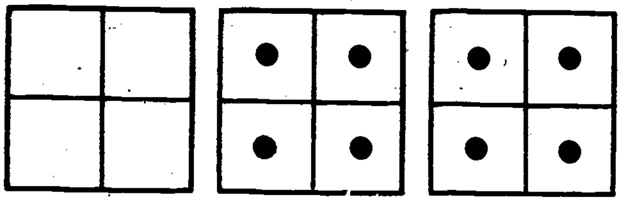
We were sometimes astonished to realize that a child who had been exceedingly weak in any knowledge of basic facts about even very small numbers, now clearly understood that, for instance, when he moved a checker from the 80 - square to the 400 - square, the new number would be 320 bigger than the old.

A careful study of the records we kept of each week's sessions helped us to discover many other interesting things that were happening in one or another of the groups. For instance, the games which are reproduced below (pages 48, 49, 50) were played just one week apart by a small group which had exactly the same students each time.

It is clear that there has been a marvelous improvement in the ability of the group, as a whole, to estimate distances more accurately in just one week's time. If one studies the first game (on pages 48 and 49), one has the feeling that the students are seeking the goal without any clear idea of where it is. The moves are small, generally tentative ones, except for one (+390) which takes them 224 above the goal, and which does inspire the two following students to be a little more brave. Then most of them revert back to very cautious steps toward the goal. In contrast, the second game (page 50) shows a steady, confident stride toward a much more distant goal, as if the students now have a better idea of its location, in relation to where they are at any given point in the game.

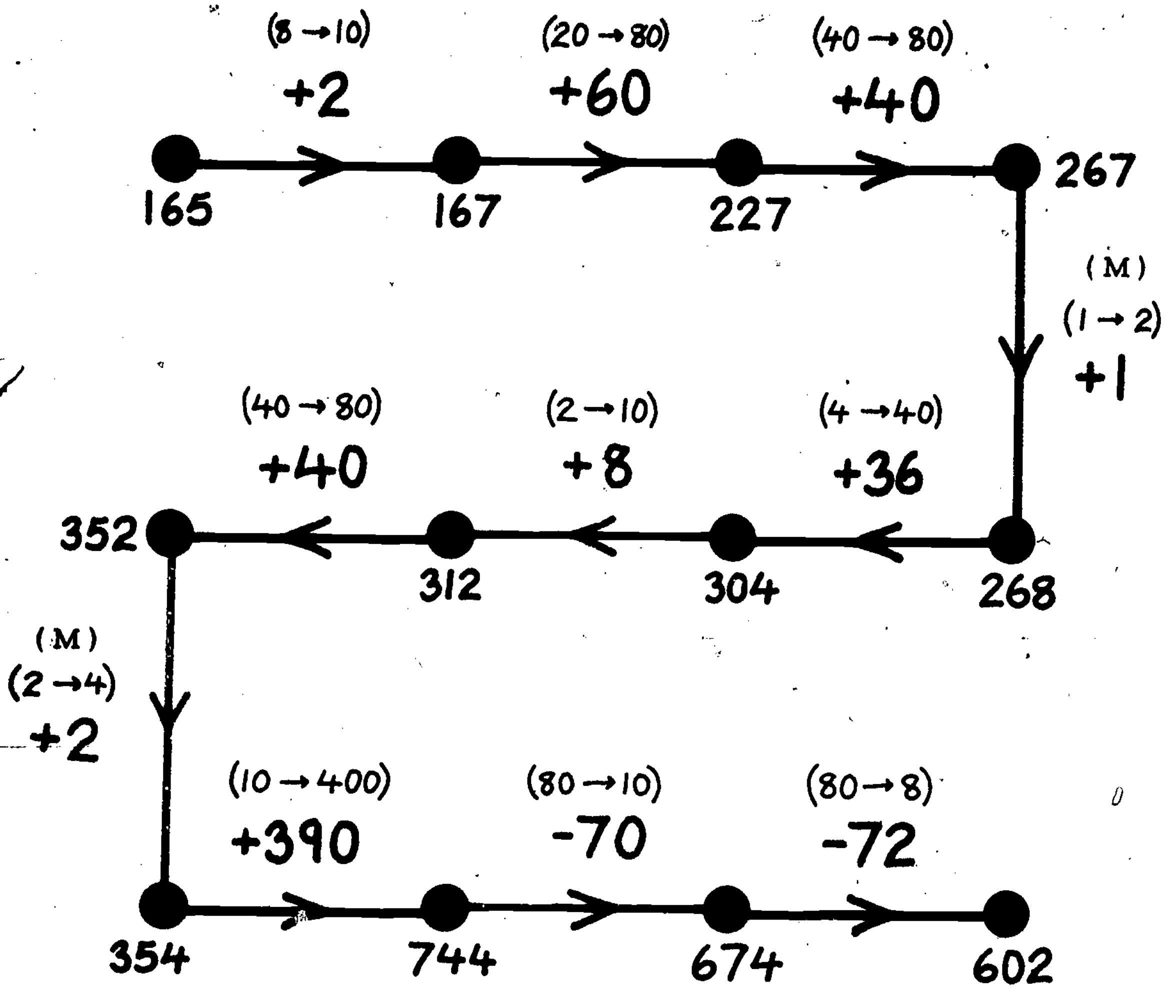
(2/3/76)

Starting boards:

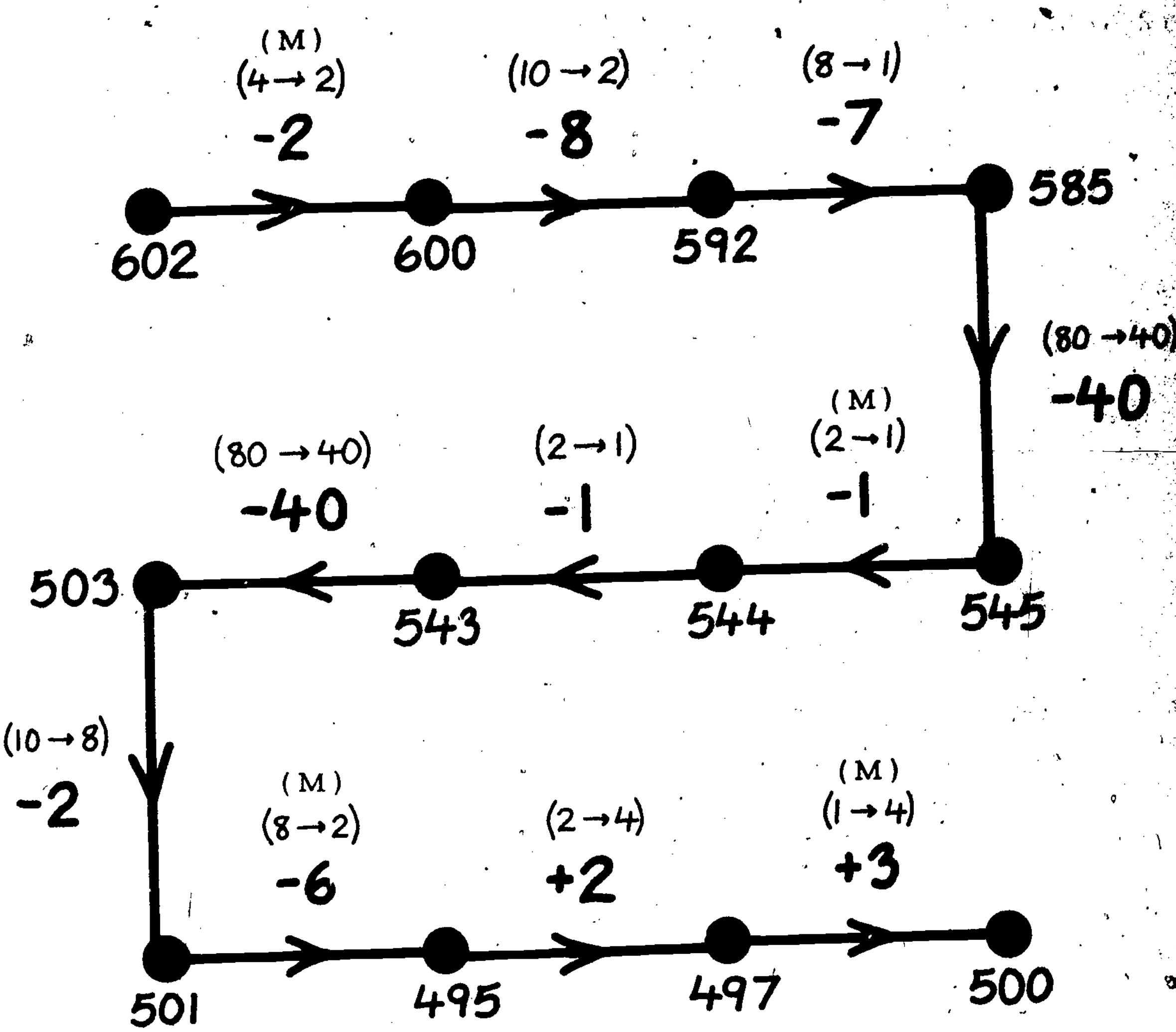


= 165

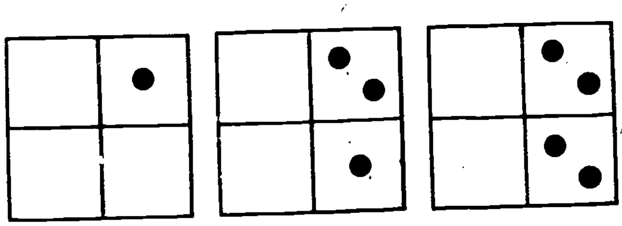
Goal: 500



(Road continues on the next page at upper left.)

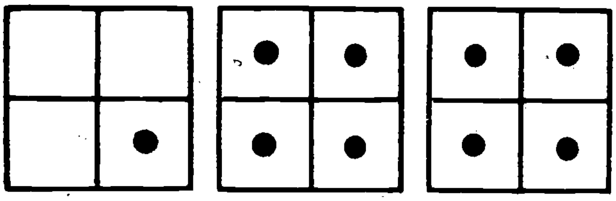


Final boards:

 = 500

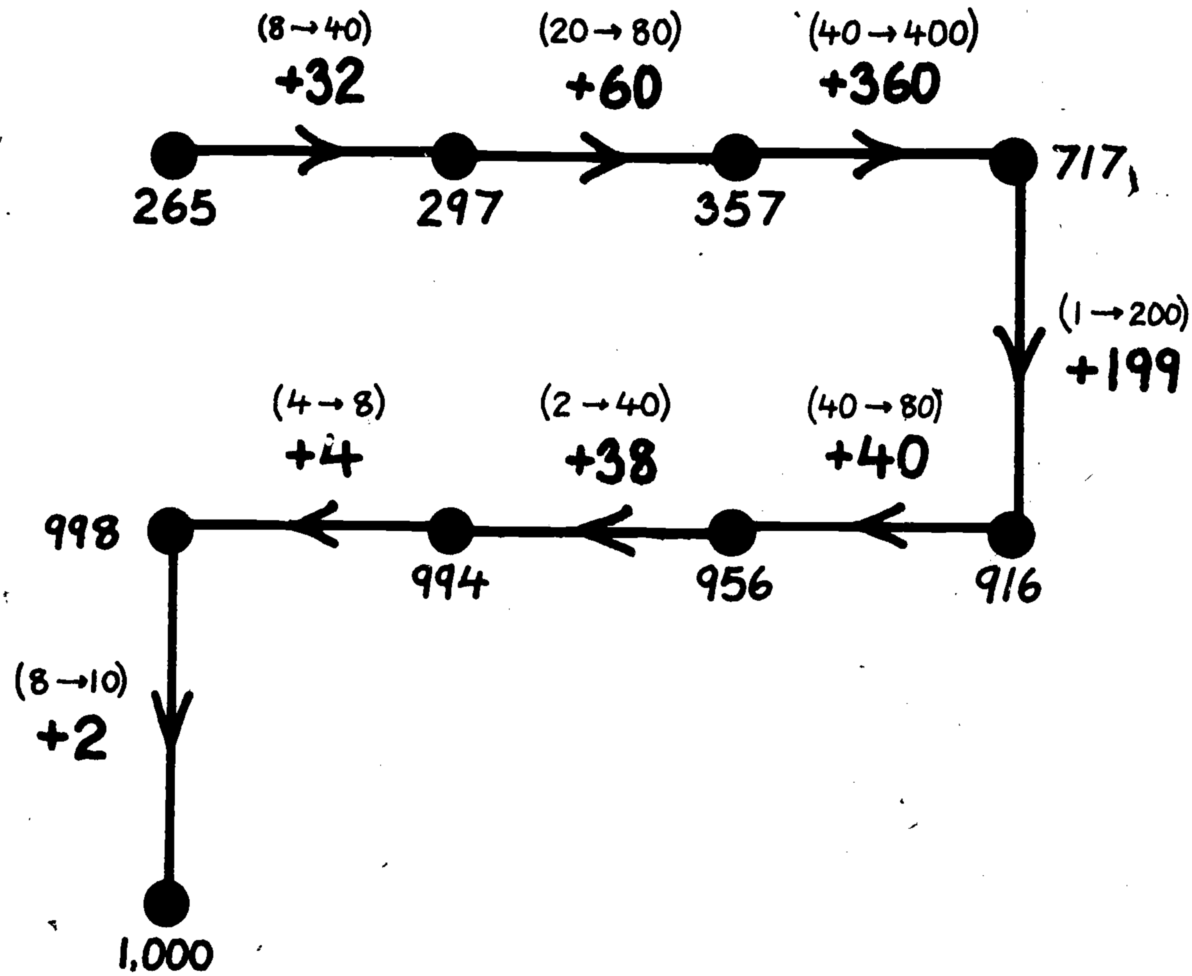
(2/10/76)

Starting boards:

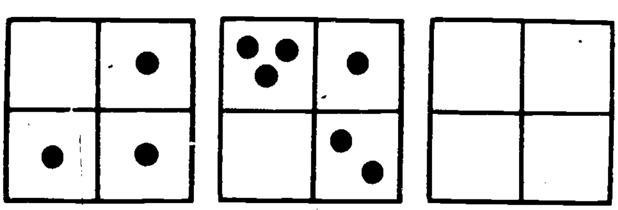


= 265

Goal: 1,000



Final boards:



= 1,000

It is interesting to study the reaction of one boy (M), who during the first game (pages 48, 49), was so concerned with the negative strategy of not making a move which allowed the other team to win that he just couldn't seem to consider the positive strategy of moving his team toward the goal. He consistently made tiny moves until the goal was close enough that he could accurately figure the distance. His last two moves were very careful ones, the second one bringing his team victory.

Another convincing illustration of the group's improvement is shown in a comparison of three other games, the first of which was played just a month later (see pages 54 - 57). These games were the first three experiences which the slow learners had in playing Minicomputer Golf using both positive and negative checkers. They were not, however, consecutive sessions. The first two were separated by one week and the second and third by five, because of various difficulties in scheduling. The overall picture clearly shows that the students were making rapid strides in developing their familiarity with the Minicomputer language and their ability to use it efficiently, even when negative checkers were involved. The teacher deliberately chose to use no more than two negative checkers and to place them carefully in order to encourage their use without creating a situation which was too difficult.

In their first game (from 137 to 400 - see pages 54, 55), it is easy to see that the accuracy of their estimation is haphazard, at best. But this sometimes turns out to be a blessing in disguise, as in the fifth move. Because of this student's great overestimation, they are involved in the use of negative checkers by necessity! However, they plunge in and use negative checkers six different times by the end of the game. So, although it is a long trip, it is not trivial or unproductive.

In their second game, two weeks later, (see page 56), they make a journey from 143 to 400. The first child makes a move which takes them well above the goal, but which also brings them much closer to it than they had been. If one sets up this game as the starting boards indicate and plays it out as the children's moves show, it is easy to see that most moves were made carefully in terms of moving an individual's team closer to the goal without playing into the hands of the other team and allowing the next opposing member to win.

Finally, after a long break because of scheduling difficulties, they played the game from 149 to 500 (see page 57). The first move is an excellent one which takes them above the goal and only 48 away from it. The second move (- 10) brings them to 538. From here there could be a victory on the third move. However, the student does not see the possibility; instead he makes a move which brings them to 518. Here again, there could be a winning move, but the next child's move makes the new number 506. And now the first child is able to make the final move and clinch the victory for his team.

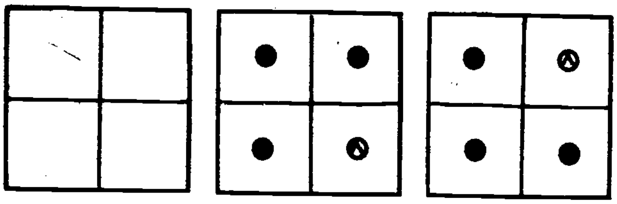
Once again we can see that the game itself is the best teacher. In a spontaneous way, it gives the students an experience in computation using negative numbers. Because they really <u>want</u> to reach that goal and win, they are forced to encounter the negative numbers and try to understand how they can use them advantageously. During the encounter, they learn about them as a by-product.

In addition, we see more evidence that learning takes place even in the interim between lessons when students have more confidence in their ability to learn. As William James observed that we learn to swim in winter and to ice skate in the summer, so we here observe that the student's understanding deepened during the month's break from small group experiences.

In our minds, there is a definite correlation between the unpressured situation in which the students were directed away from an absorption with their own inadequacies by involving them in a pleasurable, but challenging, experience with numbers and the very rewarding growth which took place. We will do further experimenting in this direction, and hope other teachers will be encouraged to do so also.

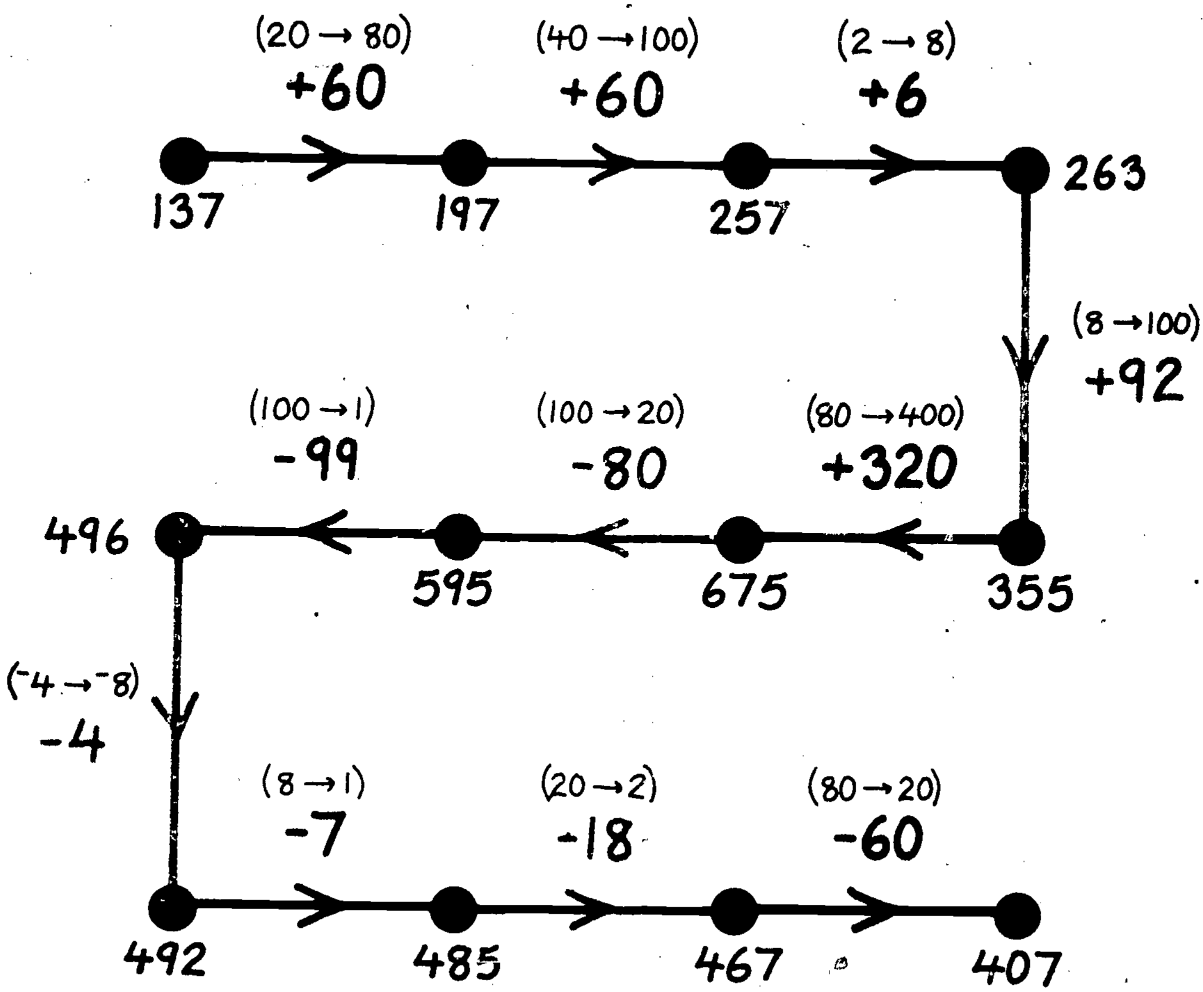
(3/9/76)

Starting boards:

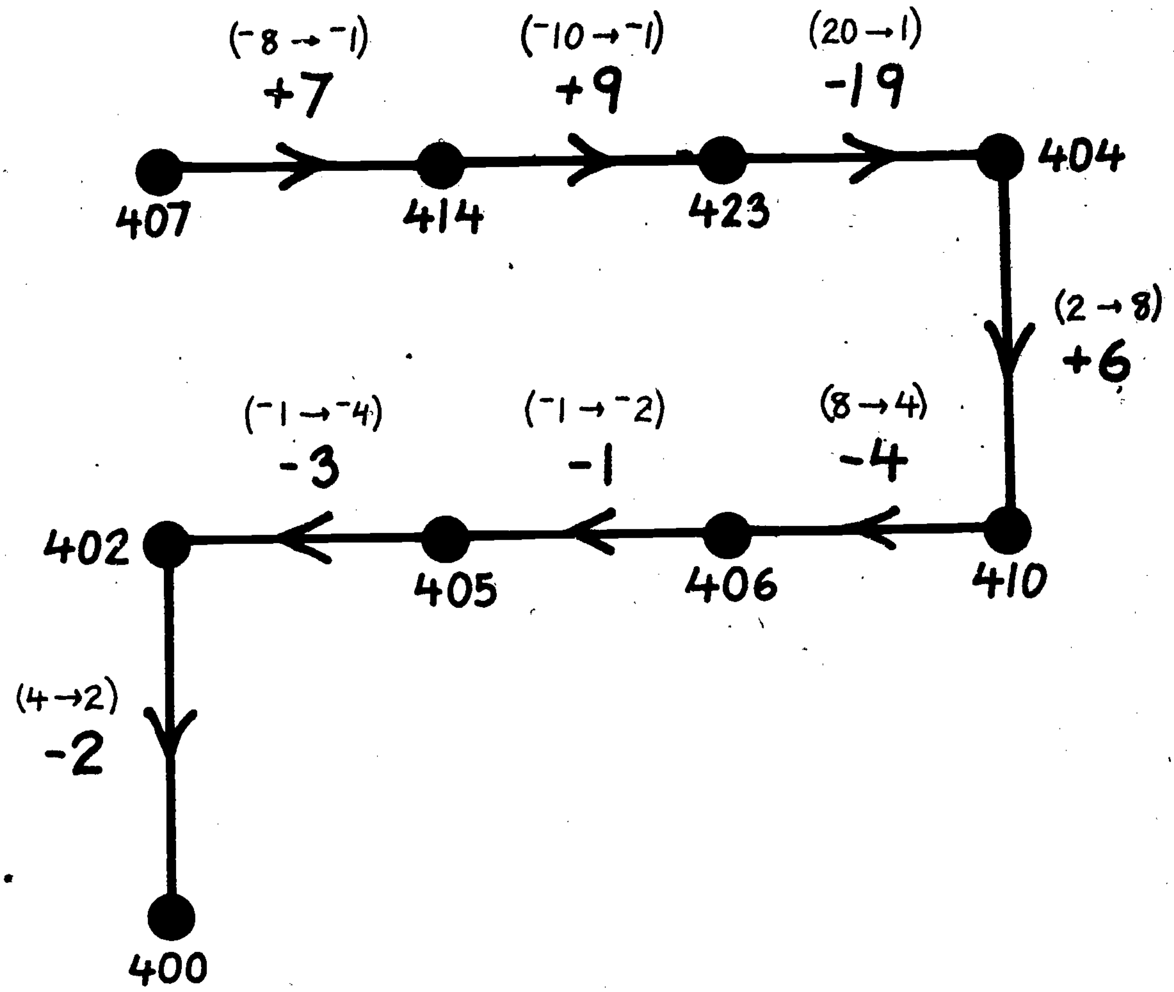


= 137

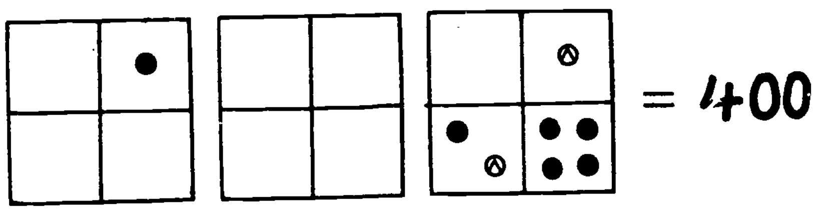
Goal: 400



(Road continues on the next page at the upper left.)

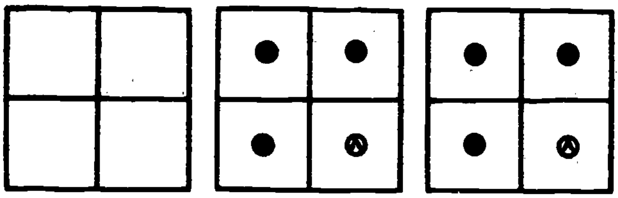


Final boards :



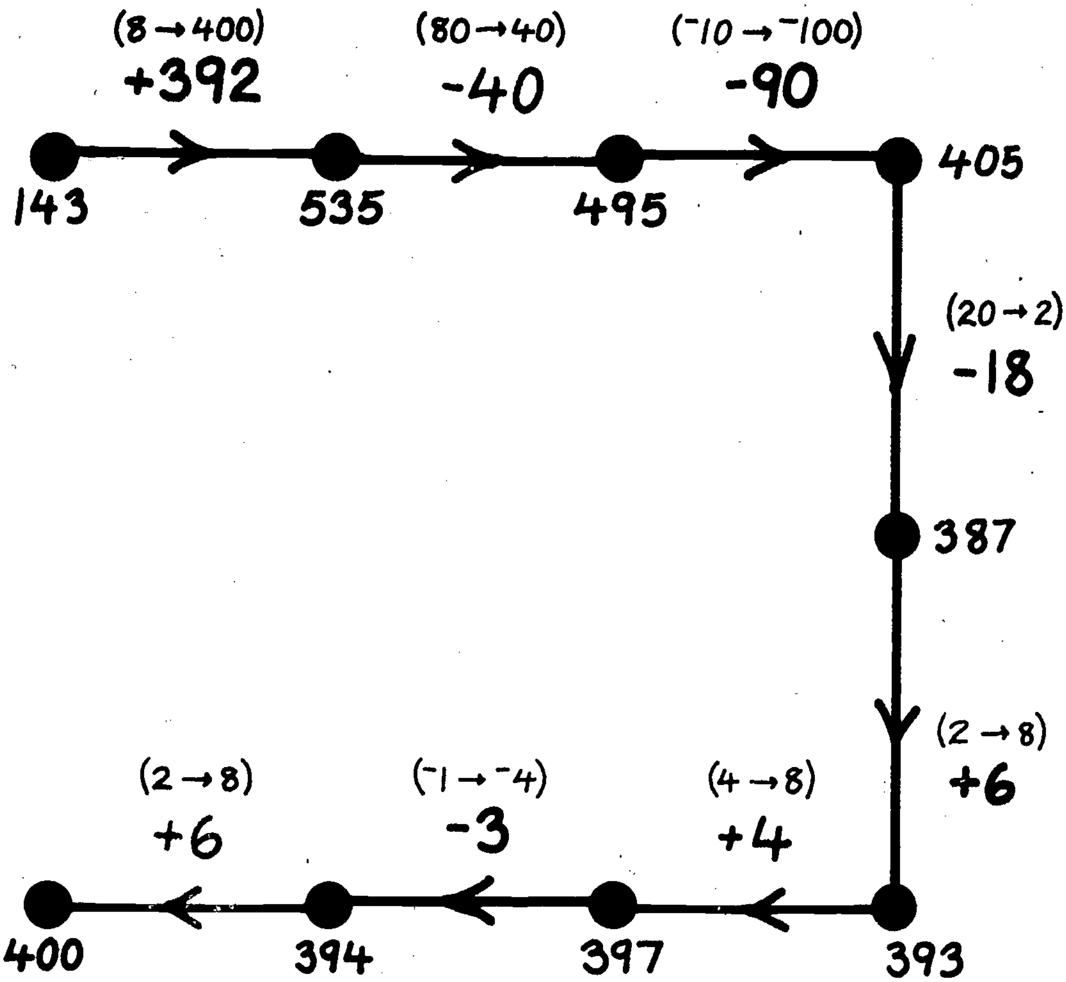
(3/23/76)

Starting boards:

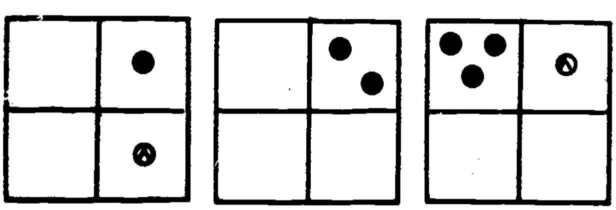


= 143

Goal: 400



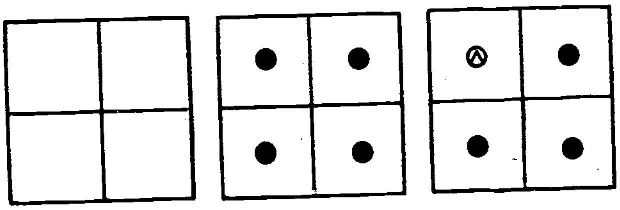
Final boards:



= 400

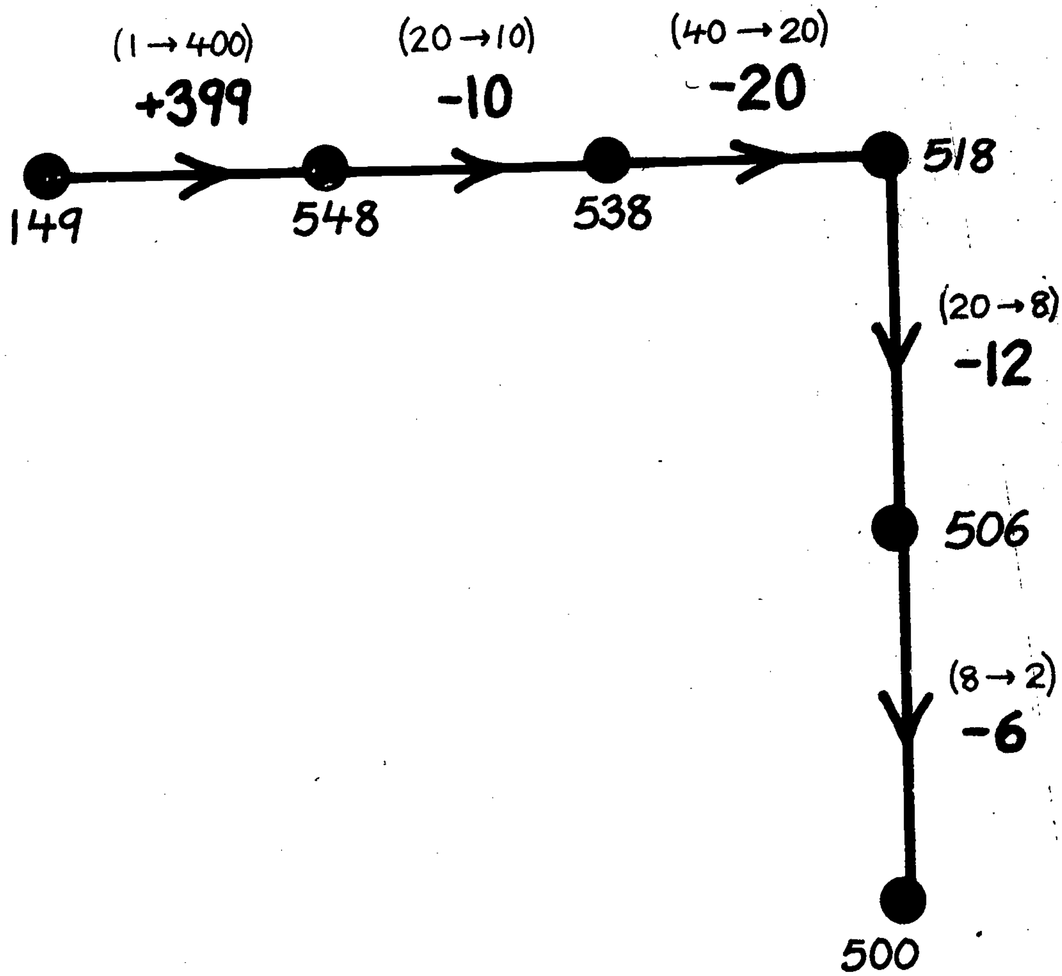
(4/29/76)

Starting boards:

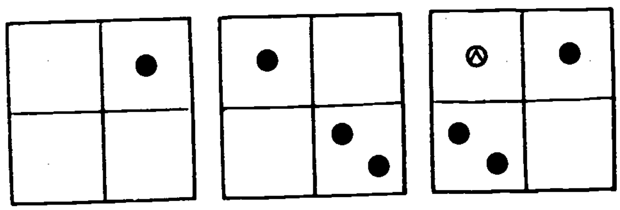


= 149

Goal: 500



Final boards:



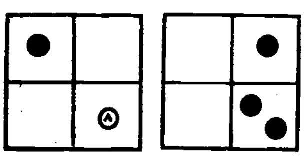
= 500

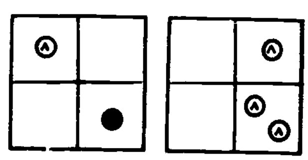
APPENDIX

The Papy Minicomputer, a kind of "paper" abacus, models the positional structure of our system of numeration and hence lends itself as a powerful tool in arithmetic calculation. It is not a sophisticated electronic device; but rather consists of one or more "boards", each board subdivided into four squares, and a set of "checkers".

When one or more Minicomputer boards is displayed, the position that each board holds relative to the other boards corresponds to place value. The values of the four squares on a board are shown on page 59: white is 1 (10, 100, etc.); red is 2 (20, 200, etc.); purple is 4 (40, 400, etc.); and brown is 8 (80, 800, etc.) Although aesthetically pleasing and pedagogically convenient, color is unnecessary and throughout this book is omitted.

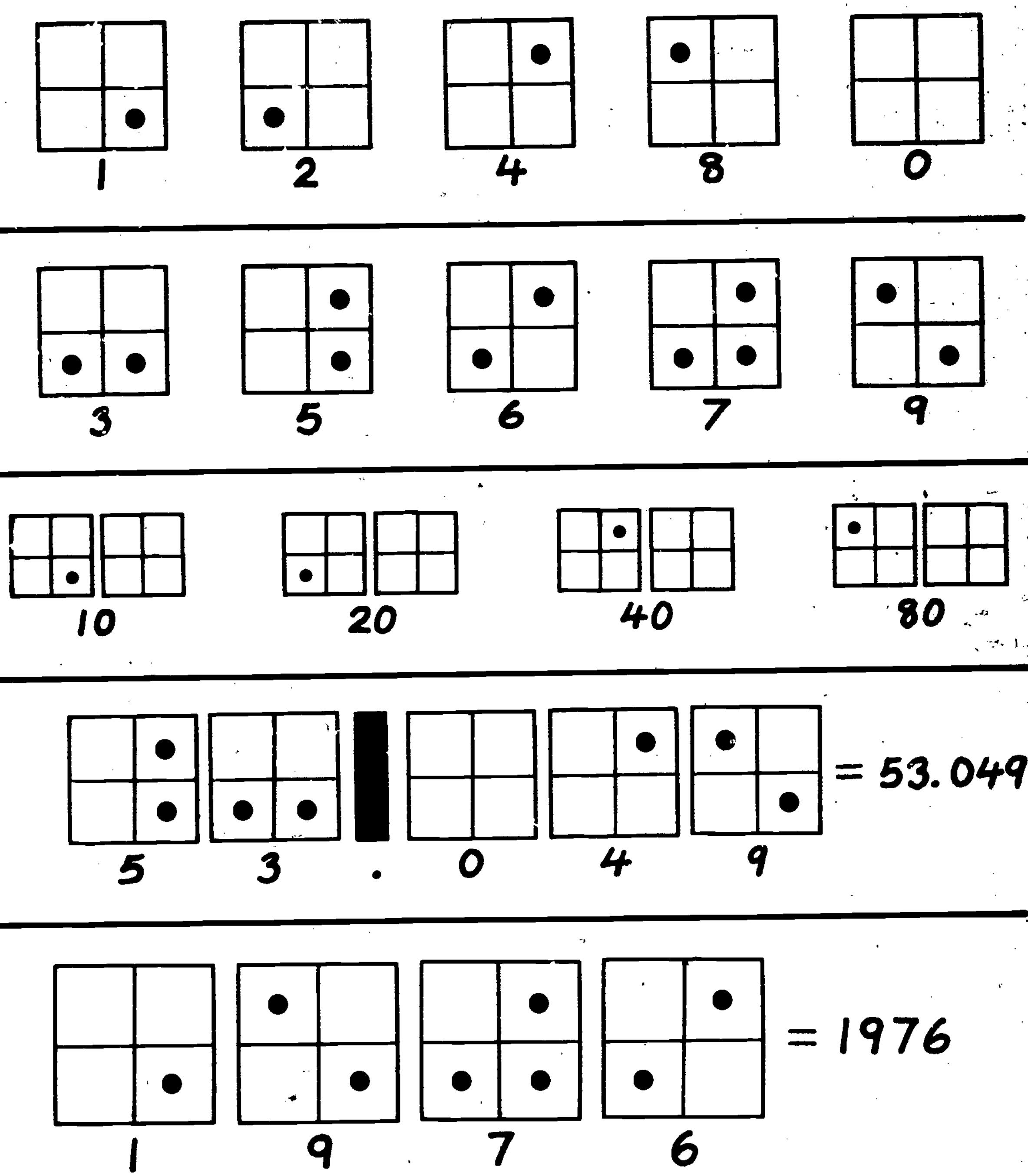
A number represented by a configuration of checkers on the Minicomputer is the sum of the values of all the checkers on the boards. Furthermore, a number has many different Minicomputer configurations. Negative numbers are represented by using checkers with "$\wedge$" written on them. For example,

 = 76

 = $^{-}76$

CSMP materials make extensive use of the Minicomputer. A list of these materials is available from CEMREL, Inc. More detailed discussion of the Minicomputer and its pedagogy can be obtained by making use of one or more of the references on page 60.

brown	purple
red	white



"Happy Twentieth Birthday, Minicomputer," by Georges Papy (St. Louis: CEMREL, Inc., 1974)

"Minicomputer," *Educational Studies in Mathematics*, pp. 333-45 (Dordrecht: D. Reidel Publishing Company, No. 2, 1969)

"Minicomputer, Un ordinateur sans electronique," *Media, No. 9*, pp. 26-36 (Paris Institute Pedagogique Nationale, January, 1970)

Minicomputer, by Georges Papy (Bruxelles: IVAC, 1969; Torino: Societa editrice internationale, 1971 (Italian))

"On Papy's Minicomputer," by Peter Braunfeld (St. Louis: CEMREL, Inc., 1974)

"Papy's Minicomputer," *Mathematics Teaching*, No. 50, Spring 1970

Summer School In The Old Days, CSMP (St. Louis: CEMREL, Inc., 1976)

"The Papy Minicomputer: A Didactical Analysis," by Peter Braunfeld (St. Louis: CEMREL, Inc., 1974)

The Papy Minicomputer: Teacher's Guide (New York: Macmillan, 1970)

Two by Two, CSMP (St. Louis: CEMREL, Inc., 1974)

3

DETECTIVE STORIES

In this chapter we present eighteen variations on the "detective story" format which was originally developed for and used in the collective lessons. Throughout the year we observed a definite correlation between our use of those stories and the deeper involvement of most of the slow learners. The stories always elicited a positive response in the collective lessons; they had the ability to engage the individual student's imagination and powers of concentration for longer periods of time.

In addition, the make-up of a detective story is particularly ideal for reviewing many concepts because it can use several or all of the mathematical languages, and in each adaptation it can present them in a slightly different environment. Therefore, each experience is new and unique, yet it is also both a review and reinforcement.

Furthermore, the technique of giving students clues to solve (introducing an element of suspense) provides them with a powerful motivation to learn to use and to become fluent in the various mathematical languages. In the same way, the element of suspense is also an inducement to follow through on a particular clue ~~which is~~ given for the hand-calculator until the number they are seeking appears. Thus, the students learn to see the hand-calculator not only as the marvelous magic box which first attracted them, but also as a very helpful tool which increases their knowledge of and ability to manipulate numbers.

Finally, the detective story enriches the material for slow learners by providing teachers with a larger variety of approaches, thus increasing the possibility that they can adapt it successfully to their own students.

The eighteen stories which we have included are just examples which teachers can feel free to alter for their own particular situations. If some clues seem too difficult, others may be substituted. Or you may prefer to create new stories from this format. This is quite acceptable and could be an interesting learning experience, in addition.

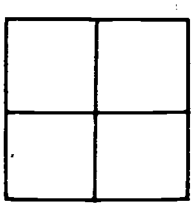
With the detective stories, as with all other activities, we believe the approach is of paramount importance. It is our hope that the teacher's attitude will be one which leads the children to relax and explore the situation with pleasure and curiosity, for we believe this is the spirit which will encourage success.

3.1 Who Is Flip? (without the help of a hand-calculator)

Our secret number is called "Flip".

First clue

Display one Minicomputer board.



T: Flip is on this Minicomputer board using exactly two (positive) checkers. You can't see Flip because the checkers are invisible.

Let the students react and show some numbers that Flip could be.
(Examples: 3; 9; 10; 4)

T: What is the smallest number that Flip could be?

S: Two.

T: What is the largest number that Flip could be?

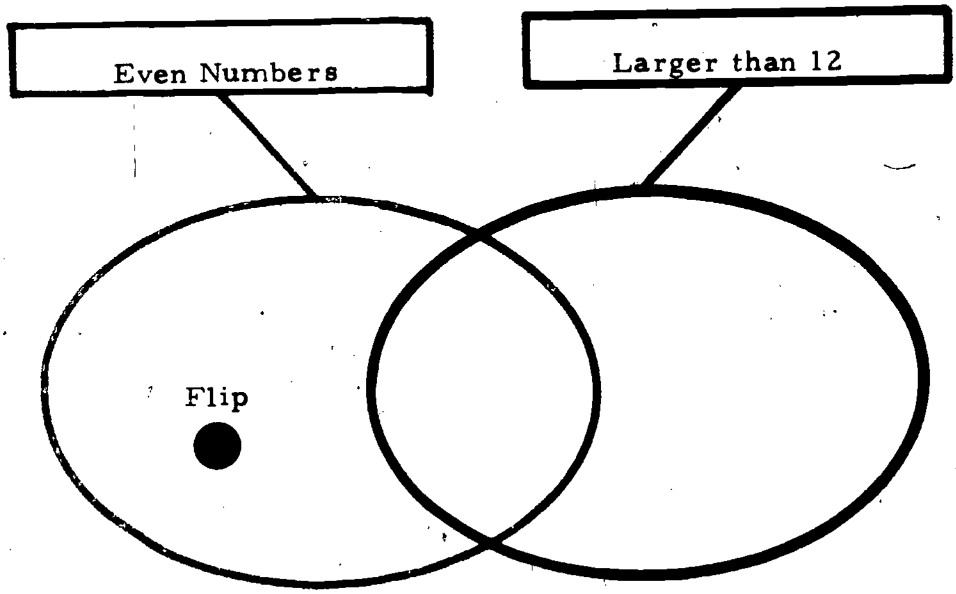
S: 16.

T: Let's make a list of all the numbers that Flip could be.

S: 2; 3; 4; 5; 6; 8; 9; 10; 12; 16.

Second clue

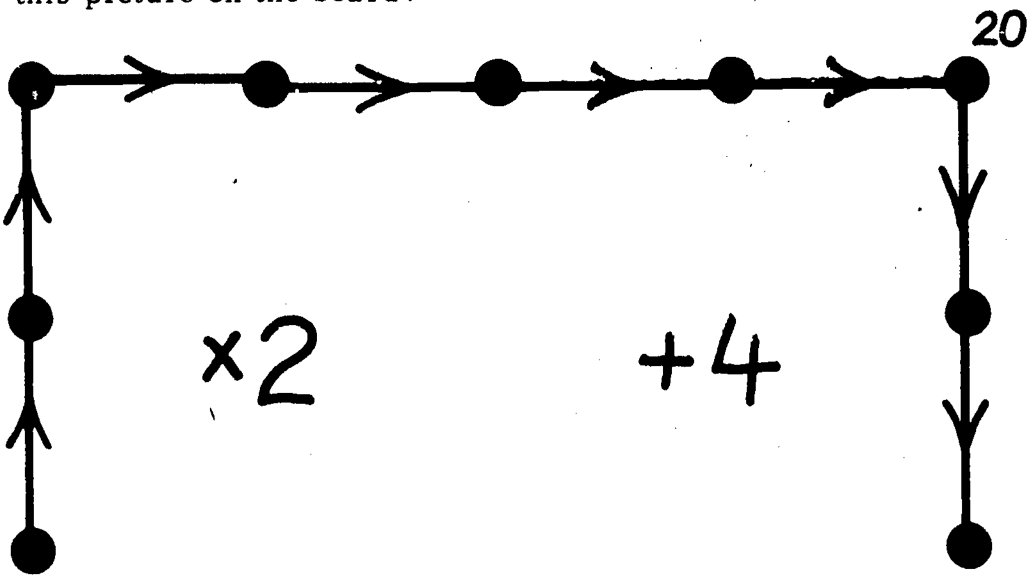
Draw this picture on the board and let the students react:



The picture shows that Flip is even and not larger than 12. So the students should conclude that Flip is one of these numbers: 2; 4; 6; 8; 10; 12.

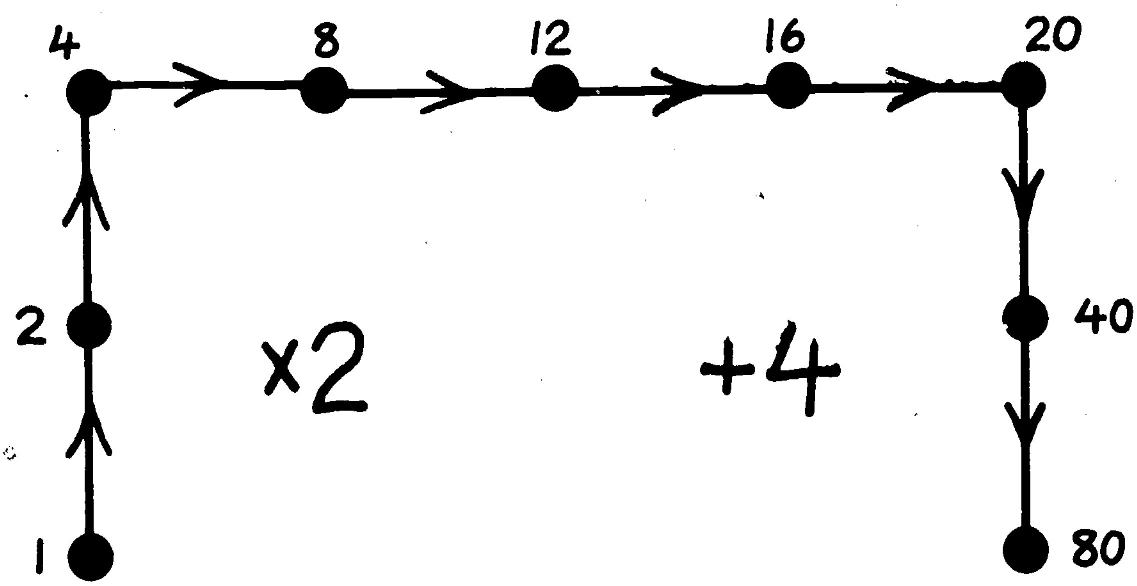
Third clue

Draw this picture on the board:



T: Flip is in this picture.

Have the students label the dots:



They should conclude that Flip is 2 or 4 or 8 or 12.

Fourth clue

T: Flip can be put on the Minicomputer with three (positive) checkers on the same square.

Ask the students to write which number Flip is on a sheet of paper.
[Answer: Flip is 12]

3.2 Who Is Kick?

Give each student a hand-calculator.

T: The name of our secret number is "Kick".

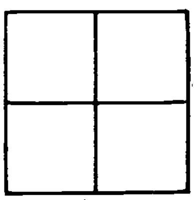
First clue

T: Start with 5 on the display.
Press $\boxed{+}$ $\boxed{2}$ $\boxed{=}$ $\boxed{=}$ and so on.
Kick will appear on the display.

Let the students observe the pattern of odd numbers. Kick can be one of these numbers: 7; 9; 11; 13; 15; 17; 19; 21; 23; . . .

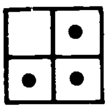
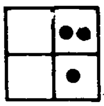
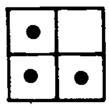
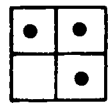
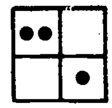
Second clue

Display one Minicomputer board.



T: Kick can be put on one board of the Minicomputer with three positive checkers.

Ask the students to put on the Minicomputer some numbers Kick could be (7; 9; 11; 13; 17) and to list some numbers Kick could not be (15; 19; 21; 23; . . .).

 = 7 ;  = 9 ;  = 11 ;  = 13 ; AND  = 17

They should conclude that Kick is 7 or 9 or 11 or 13 or 17.

Third clue

T: Start with 387 on the display.

Press $\boxed{-}$ $\boxed{1}$ $\boxed{0}$ $\boxed{=}$ $\boxed{=}$ and so on.

Kick will appear on the display.

Let the students observe that the last digit of each number that appears on the display is 7. They should conclude that Kick is 7 or 17.

Fourth clue

T: Do you remember what a square number is?

Can you give me examples of square numbers? (4 = 2 x 2; 16 = 4 x 4; 9 = 3 x 3; etc.)

Can you give me examples of numbers that are not square numbers? (2; 3; 5; 6; 7; 8; 10; 11; 12; 13; 14; 15; 17; 18; . . .)

Now for the last clue.

If you subtract 1 from Kick, you will get a square number.

After the students try each number for Kick they should conclude that Kick is 17 because 17 - 1 = 16 = 4 x 4, and 7 - 1 = 6 is not a square number.

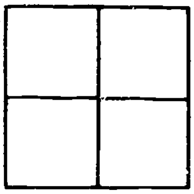
3.3 Who Is Krack?

Give each student a hand-calculator.

T: The name of our secret number is "Krack".

First clue

Display one Minicomputer board.



T: Krack can be put on one Minicomputer board with exactly two positive checkers.

Let the students show on the Minicomputer that Krack is one of these numbers: 2; 3; 4; 5; 6; 8; 9; 10; 12; 16.

Second clue

T: If I put Krack on the display of my hand-calculator and press $\boxed{+}$ $\boxed{=}$ $\boxed{=}$ and so on, 72 will appear on the display.

Let the students use a hand-calculator to see which numbers Krack could be. It is most likely that some students, without the use of the hand-calculator, will be able to tell you that Krack cannot be 5 or 10. This response should be encouraged when the opportunity presents itself. The students should conclude that Krack is one of these numbers: 2; 3; 4; 6; 8; 9; 12.

Third clue

T: If I put Krack on the display of my hand-calculator and press $\boxed{+}$ $\boxed{=}$ $\boxed{=}$ and so on, 81 will appear on the display.

Let the students try the numbers that Krack could be. With little or no use of the hand-calculator, some students may conclude that Krack cannot be 2 ; 4 ; 8 ; or 12 since they are even. The students should conclude that Krack is 3 or 9.

Fourth clue

T: If I put Krack on the display of my hand-calculator and press $\boxed{+}$ $\boxed{=}$ $\boxed{=}$ and so on, 102 will appear on the display.

Encourage the students to eliminate 9 without the use of the hand-calculator. For example:

T: Start with 9 on the display. Hide the display of your calculator. Press $\boxed{+}$ and then $\boxed{=}$ ten times. Can you predict what number will appear on the display?

S: 90.

T: Check your prediction. If you continue pressing $\boxed{=}$, will 102 appear on the display?

S: No, 99 will appear and so will 108, but not 102.

T: Now start with 3 on the display. Hide the display with one hand. Press $\boxed{+}$ and then $\boxed{=}$ ten times. Can you predict what number will appear on the display?

S: 30.

T: Check your prediction. Now suppose I press $\boxed{=}$ 20 more times. What number will appear?

S: 90.

T: If you continue pressing $\boxed{=}$, will 102 appear on the display?

S: Yes.

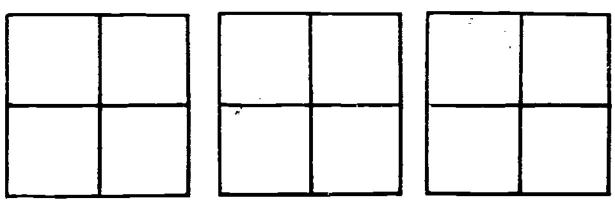
The students should conclude that Krack is 3.

3.4 <u>Who Is Zot?</u> (without the help of a hand-calculator)

T: Our secret number is called "Zot".

<u>First clue</u>

Display three Minicomputer boards.



T: Zot is on these Minicomputer boards using exactly two (positive) checkers on the same square. You can't see Zot because the checkers are invisible.

Let the students react to the clue and put some numbers Zot could be on the Minicomputer. Suggest that the students look for a pattern.

T: What is the smallest number that Zot could be?

S: Two.

T: What is the largest number that Zot could be?

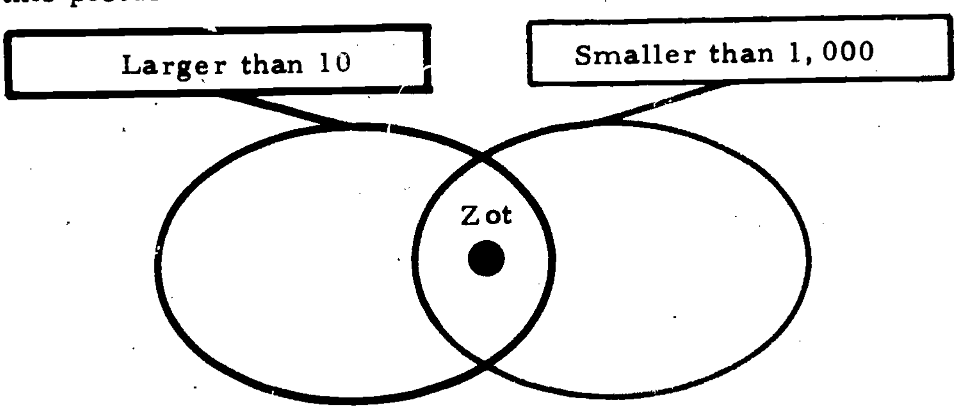
S: 1,600.

T: Let's make a list of all the numbers that Zot could be.

A careful listing of the numbers that Zot could be will stress the pattern. [Answer: 2; 20; 200; 4; 40; 400; 8; 80; 800; 16; 160; 1,600]

<u>Second clue</u>

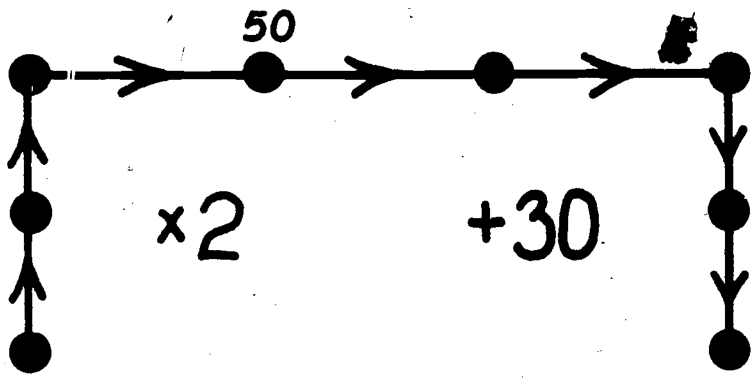
Draw this picture on the board and let the students react to it:



The students should conclude that Zot is one of the following numbers: 16; 20; 40; 80; 160; 200; 400; 800.

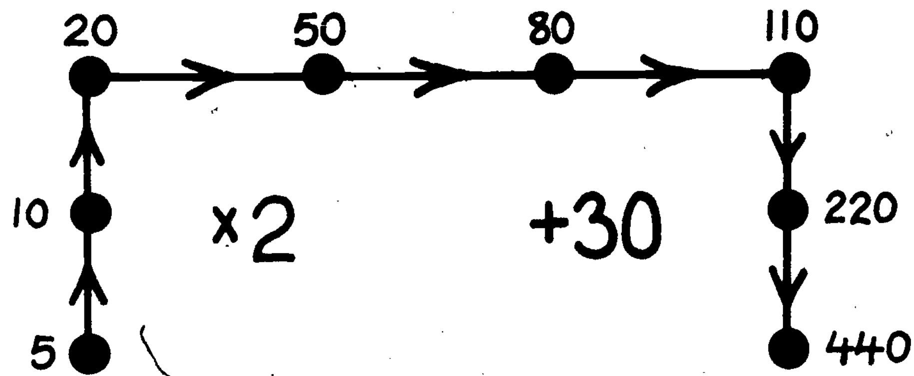
Third clue

Draw this picture on the board:



T: Zot is in this arrow picture.

Have the students label the dots:



They should conclude that Zot is 20 or 80.

Fourth clue

T: Zot can be put on the Minicomputer with four (positive) checkers on the same square.

The students should conclude that Zot is 80.

3.5 Who Is Kwa?

Give each student a hand-calculator.

T: The name of our secret number is "Kwa".

First clue

T: Press $\boxed{2}$ $\boxed{\times}$ $\boxed{=}$ $\boxed{=}$ and so on. Kwa will appear on the display. (NOTE: Examine the hand-calculators that you will use with this story. Some calculators will not permit multiplication by a constant in the manner suggested above. If your calculators are of this type, substitute " $\boxed{1}$ $\boxed{\times}$ $\boxed{2}$ $\boxed{=}$ $\boxed{=}$ . . . " for " $\boxed{2}$ $\boxed{\times}$ $\boxed{=}$ $\boxed{=}$ . . . ")

Let the students observe the numbers as they appear on the display.

T: Cover the display of your calculator with one hand.
Press $\boxed{2}$ $\boxed{\times}$ $\boxed{=}$ $\boxed{=}$
Can you predict what number will be on the display?

S: 8.

T: Now press $\boxed{=}$ again and make another prediction.

S: 16.

T: Uncover the display to check your prediction. If you press $\boxed{=}$ again, what number will be on the display?

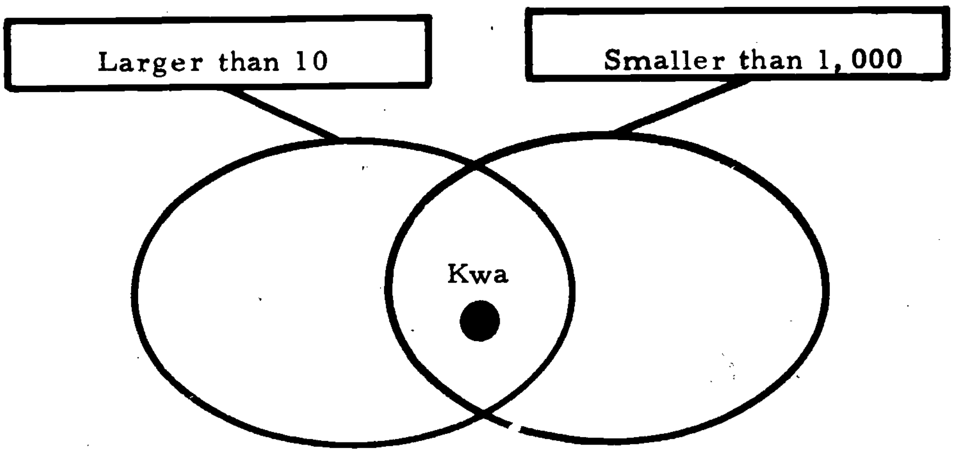
S: 32.

If necessary, this line of questioning may be continued. Kwa can be:

2; 4; 8; 16; 32; 64; 128; 256; . . .

Second clue

Draw this picture on the board:



T: Kwa is in this string picture.

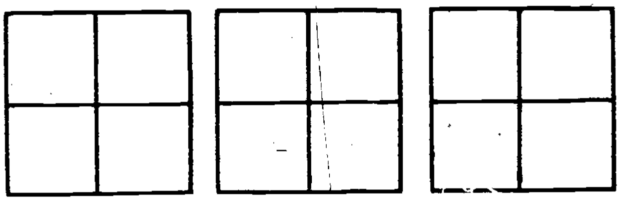
Encourage the students to verbalize the new information about Kwa. (Kwa is larger than 10 and smaller than 1,000.)

T: Let's list numbers Kwa can be. 2? (No); 4? (No); 8? (No); 16? (Yes); 32? (Yes); 64? (Yes); 128? (Yes); 256? (Yes); 512? (Yes); 1,024? (No)

The students should conclude that Kwa is one of the following numbers: 16; 32; 64; 128; 256; 512.

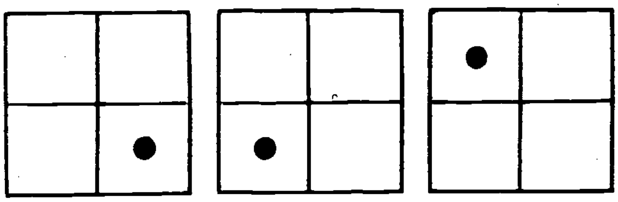
Third clue

Display three Minicomputer boards.



T: Kwa can be put on the Minicomputer using exactly one positive checker on *each* board.

Let the students try putting numbers from the above list on the Minicomputer. They will notice that 128 is the only number that meets the requirements of the third clue.

128 = 

The students should conclude that Kwa is 128.

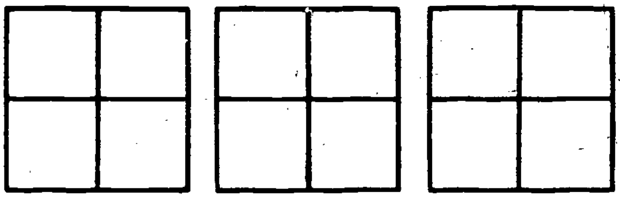
3.6 Who Is Kong?

Give each student a hand-calculator.

T: The name of our secret number is "Kong".

First clue

Display three Minicomputer boards.



T: Kong can be put on the Minicomputer with exactly two positive checkers.

Give the students plenty of time to tell numbers that Kong could be. Since there are 76 possibilities, the students should not be expected to give a complete list of these numbers.

After a while, you could challenge the students with some questions. For instance,

T: Could Kong be 28? (Yes) Show us.
What about 20? (Yes); 10? (Yes); 204? (Yes); 409? (No); 100? (Yes); 1,000? (Yes)
What is the smallest number Kong could be?

S: 2.

T: The largest?

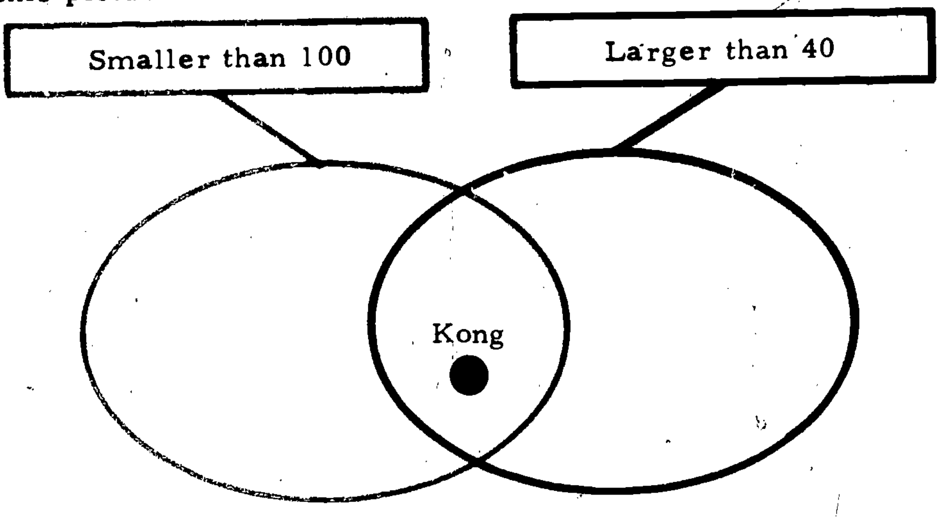
S: 1,600.

T: What is the largest two digit number Kong could be?

S: 90.

Second clue

Draw this picture on the board:



T: Kong is in this string picture.

Let the students play with the clue. Encourage them to verbalize the new information about Kong. (Kong is smaller than 100 and larger than 40.)

T: Could Kong be 40?

S: No, Kong is larger than 40.

T: 41?

S: Yes.

T: 45?

S: No, 45 cannot be put on the Minicomputer with two positive checkers.

T: How about 48?

S: Yes.

T: 50?

S: Yes.

T: How about 51 through 59?

S: None of them can be put on the Minicomputer with two positive checkers.

If necessary, this line of questioning can be continued. The students should conclude that Kong is 88 or 48.

Fourth clue

T: Do you know what a square number is?

Give some examples of square numbers ($3 \times 3 = 9$; $5 \times 5 = 25$; $10 \times 10 = 100$; $4 \times 4 = 16$; and so on), and some examples of numbers which are not square (5; 8; 15; and so on).

T: Now my last clue is: If you add 1 to Kong, you will get a square number.

The students should conclude that Kong is 48 because $48 + 1 = 49 = 7 \times 7$, while $88 + 1 = 89$ is not a square number.

3.7 Who Is Nim?

Give each student a hand-calculator.

T: The secret number is called "Nim".

First clue

T: Start with 53 displayed on your calculator.
Then press $\boxed{+}$ $\boxed{1}$ $\boxed{0}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ and so on.
Nim will appear on the display.

First let the students react to this clue and watch the numbers that appear on the display. Encourage them to express their ideas about Nim. Then challenge them with some questions. For instance,

T: Could Nim be 216?

S: No.

T: Why not?

S: Because the last digit is not "3".

T: Could Nim be 23?

S: No.

T: Why not?

S: Because Nim is larger than 53.

T: Could Nim be larger than 1,000?

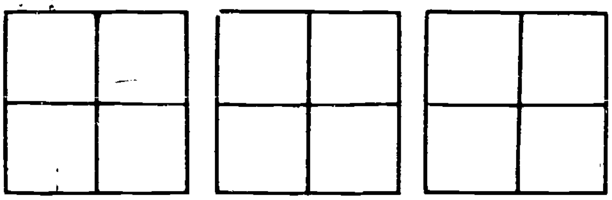
S: Yes.

T: Give some examples of numbers larger than 1,000 that Nim could be.

S: 1,003; 1,013; 1,023; 1,033; . . . ; 6,793; . . . ; 153,283; . . .

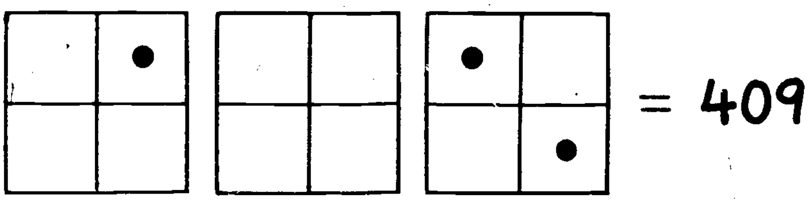
<u>Second clue</u>

Display three Minicomputer boards.



T: Nim can be put on the Minicomputer with one positive checker on the hundreds' board and two positive checkers on the ones' board.

Ask students to put on the Minicomputer numbers that Nim could be. It is likely that you will get some wrong answers. For instance,



In ~~this case~~, let the students react and explain that Nim cannot be 409 because the last digit of "409" is not "3". (See first clue)

After some tries, the students should conclude that Nim is 103 or 203 or 403 or 803.

Third clue

T: Clear your display and press $\boxed{+}$ $\boxed{7}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ and so on. Nim will appear on the display.

Let the students work on their own for a while. They will probably notice that 103 doesn't appear on the display while 203 does. At this point you could ask the students to turn their hand-calculator off.

T: Now we know that Nim could not be 103, but that he could be 203. Could Nim be 403? Try to answer this question without the help of your hand-calculator.

If the suggestions of the students are not articulate enough, you could help them by asking:

T: If we continue pressing $\boxed{=}$, the number 203 + 203 = 406 will appear on the display. What about 403?

The students should notice that 406 - 7 = 399 will appear on the display, but not 403. So Nim could not be 403.

T: If we press $\boxed{+}$ $\boxed{7}$ and then $\boxed{=}$ 100 times, what number will appear on the display? (100 x 7 = 700)
Suppose we press $\boxed{=}$ 10 more times. What number will appear on the display? (770)
Suppose we continue pressing $\boxed{=}$. What numbers will appear on the display? (777; 784; 791; 798; 805; . . .)

So the students will notice that 803 will not appear on the display. They should conclude that Nim is 203.

3.8 Who Is Tack?

Give each student a hand-calculator.

T: Tack is a secret <u>whole</u> number.

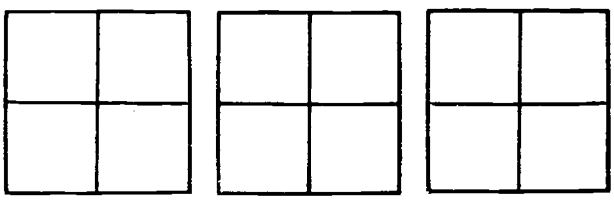
First clue

T: If I put Tack on the display of my hand-calculator and press $\boxed{+}\ \boxed{3}\ \boxed{\times}\ \boxed{1}\ \boxed{0}\ \boxed{=}$, the number that appears on the display is between 200 and 300.

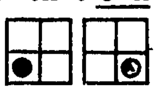
Let the students have time to try some numbers for Tack. They should conclude that Tack is one of the following numbers: 18; 19; 20; 21; 22; 23; 24; 25; 26.

Second clue

Display three Minicomputer boards.



T: Tack <u>cannot</u> be put on the Minicomputer using exactly two checkers (positive or negative).

First ask the students to show some numbers that <u>can</u> be put on the Minicomputer with exactly two checkers. (18;  = 19; 20; 21; 22; 24) Then the students should conclude that Tack is 23, 25 or 26.

Third clue

T: Do you remember what a square number is?

Can you give me examples of square numbers? ($4 = 2 \times 2$; $1 = 1 \times 1$; $16 = 4 \times 4$; etc.)

Can you give me examples of numbers that are not square? (2; 3; 5; 6; 7; 8; 10; etc.)

Now I will give you the third clue.

Tack is not a square number.

The students should conclude that Tack is 23 or 26.

Fourth clue

T: Start with 800 on the display.

Press $\boxed{-}$ $\boxed{7}$ $\boxed{=}$ $\boxed{=}$ and so on.

Tack will appear on the display.

Let the students play with this clue. After a while, continue with

T: Put your calculators aside. Suppose I start from 800 and press $\boxed{-}$ $\boxed{7}$ and then $\boxed{=}$ 100 times. Can you predict what number will appear?

S: 100.

T: Why?

S: $100 = 800 - (100 \times 7)$

T: Now we know that 100 will appear on the display.

T: Start from 100. Hide the display of your calculator with one hand.
Press $\boxed{-}$ $\boxed{7}$ and then $\boxed{=}$ 10 times.
Can you predict what number will appear on the display?

S: 30.

T: Why?

S: $30 = 100 - (10 \times 7)$

T: Check your prediction. Will 26 appear?

S: No.

T: Will 23 appear?

S: Yes.

T: Why?

S: $23 = 30 - 7$

The students should conclude that Tack is 23.

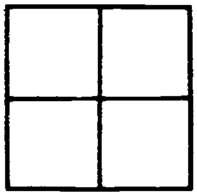
3.9. Who Is Kim?

Give each student a hand-calculator.

T: The secret number is called "Kim".

First clue

Display one Minicomputer board.



T: Kim can be put on the ones' board with one positive checker and one negative checker.

Ask the students to tell some of the numbers Kim could be. For example, 3 and $^{-}3$; 7 and $^{-}7$.

T: What is the largest number that Kim could be?

S: 7.

T: What is the smallest number that Kim could be?

S: $^{-}7$.

T: Let's make a list of all the numbers that Kim could be.

Ask the students to put on the Minicomputer the numbers that Kim could be. [Answer: 7; 6; 4; 3; 2; 1; 0; $^{-}1$; $^{-}2$; $^{-}3$; $^{-}4$; $^{-}6$; $^{-}7$]

Second clue

T: Start with 2, 374 displayed on your calculator.
Press [-] [1] [0] [=] [=] and so on.
Kim will appear on the display.

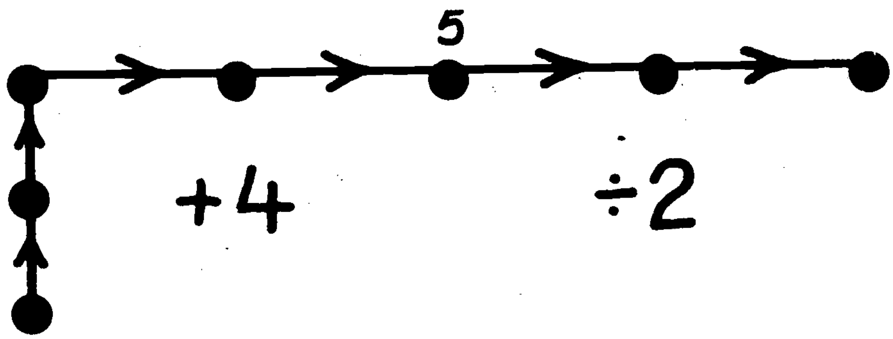
Let the students work on their own for a while and watch the numbers that appear on the display. Challenge them with some questions. For instance,

T: If you keep pressing $\boxed{=}$, what is the first number smaller than 2,000 to appear on the display? (1,994); the first number smaller than 1,000? (994); the smallest positive number? (4); the first negative number? ($^-6$); the first negative number smaller than $^-100$? ($^-106$)

What numbers could Kim be? (4 or $^-6$)

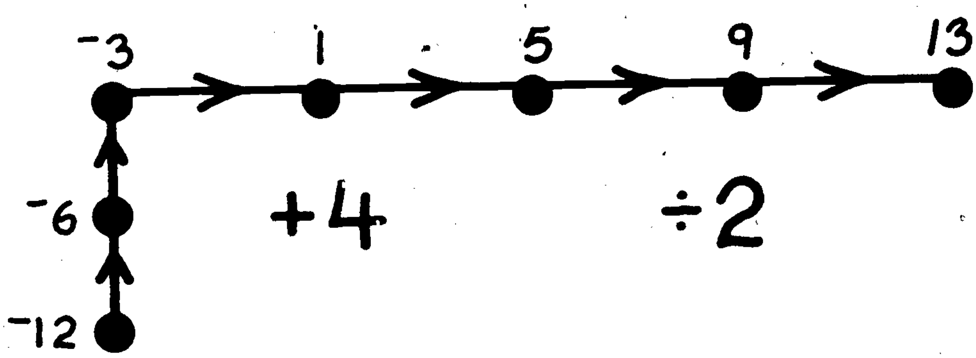
Third clue

Draw this picture on the board:



T: Kim is in this arrow picture.

Have the students label the dots:



The students should conclude that Kim is $^{-}6$.

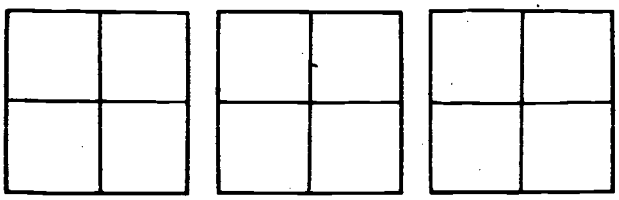
3.10 Who Is Pim?

Give each student a hand-calculator.

T: The name of our secret number is "Pim".

First clue

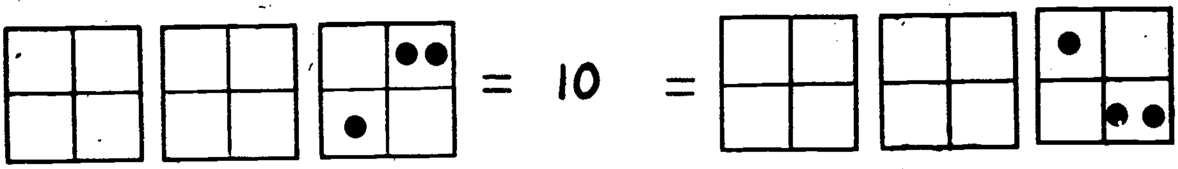
Display three Minicomputer boards.

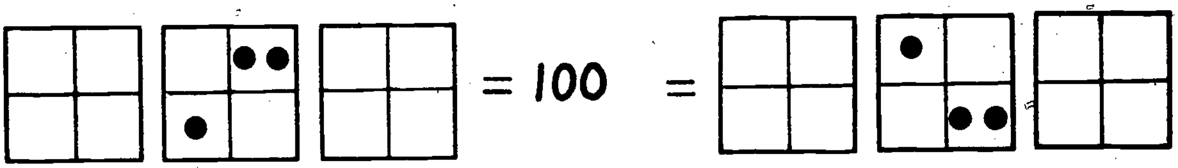


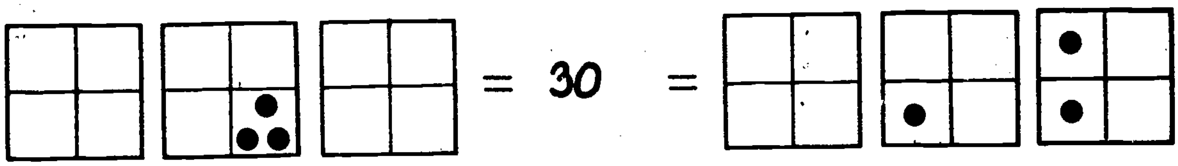
T: Pim can be put on the Minicomputer with exactly three positive checkers.

Let the students have plenty of time to determine numbers that Pim could be. It is obvious that the students will not list all of the numbers that Pim could be. You may want to ask questions like the following:

T: Could Pim be 10? (Yes) Show us.
100? (Yes) 102? (Yes) 87? (No)
Could Pim be 30? (Yes) 300? (Yes)







Even though a number may have several configurations, it is not necessary for the student to show all of them.

T: What is the largest number that Pim could be? (2, 400)

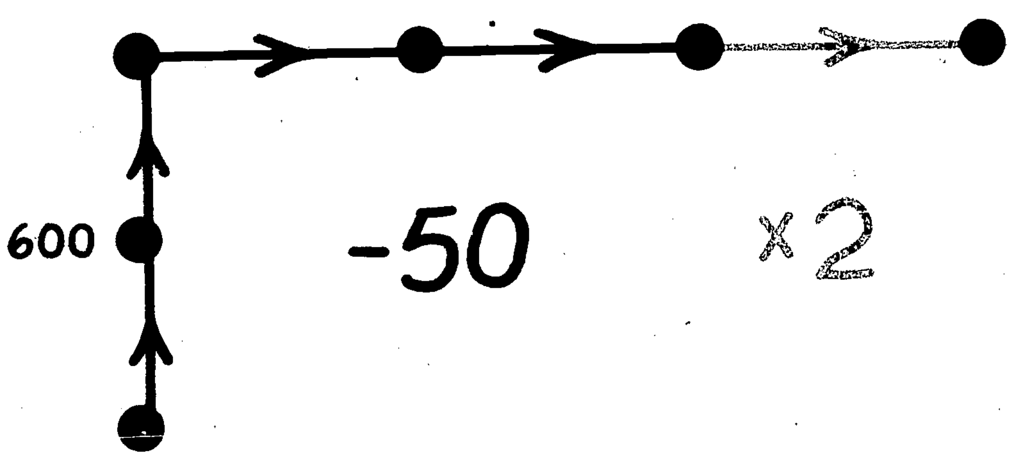
What is the smallest? (3)

Can you name some other numbers Pim could be?

Can you name some numbers Pim could not be?

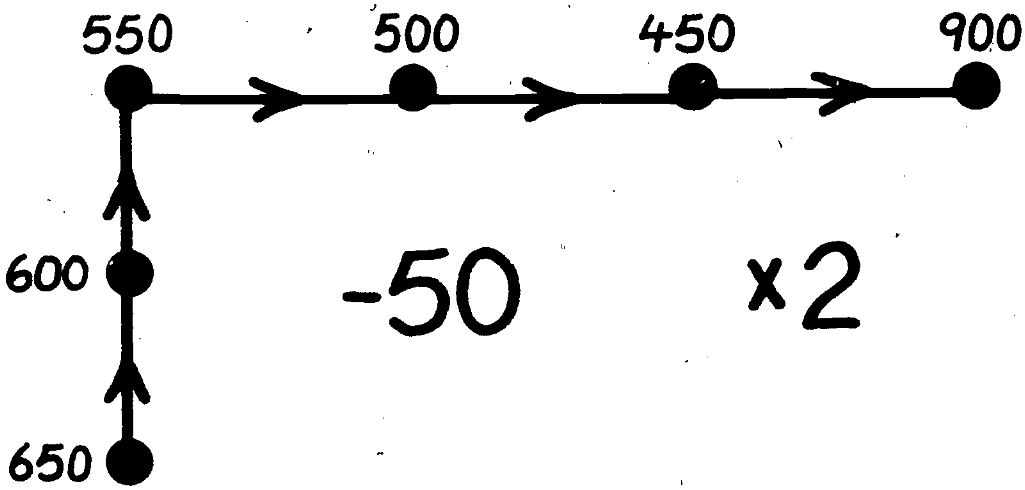
<u>Second clue</u>

Draw this picture on the board:



T: Pim is in this arrow picture.

Let the students label the dots :



T: Don't forget that Pim can be put on the Minicomputer with exactly three positive checkers.

What numbers could Pim be? (600 or 500 or 450 or 900)

Third clue

T: Press $\boxed{+}$ $\boxed{3}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ and so on.

Pim will appear on the display.

Let the students observe the numbers that appear on the display. Encourage them to verbalize any ideas they have.

T: Will 30 appear on the display?

S: Yes.

T: Why?

S: $30 = 10 \times 3$; 30 is a multiple of 3.

T: Will 60 appear?

S: Yes, 60 = 20 x 3.

T: Will 90 appear?

S: 90 = 30 x 3.

T: Will 150 appear?

S: Yes, 150 = 50 x 3.

T: Will 300 appear?

S: Yes, 300 = 100 x 3.

T: Will 450 appear?

S: Yes, 450 = 300 + 150 = 150 x 3.

T: Will 500 appear?

S: No.

Starting from 450 and pressing $\boxed{+}$ $\boxed{3}$ $\boxed{=}$ $\boxed{=}$ and so on, let the students observe the numbers that appear on the display.

T: Will 600 appear?

S: Yes, 600 = 200 x 3.

T: Will 900 appear?

S: Yes, 900 = 300 x 3.

The students should conclude that Pim is 450 or 600 or 900.

Fourth clue

T: Do you remember what a square number is?
Give us some examples of square numbers. (4 = 2 x 2; 25 = 5 x 5; 1 = 1 x 1; 49 = 7 x 7; 100 = 10 x 10; . . .).

With the help of your hand-calculator, try to make a list of all the square numbers between 100 and 1,000.

When the students have completed this list, give them the fourth clue.

T: Pim is a square number.

The students should conclude that Pim is 900.

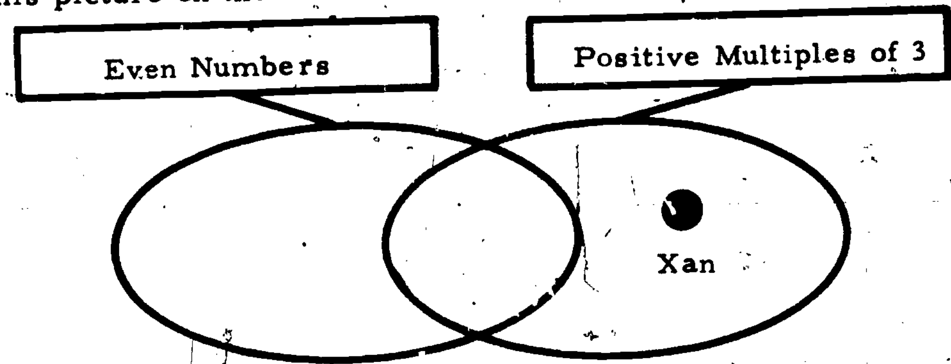
3.11 Who Is Xan?

Give each student a hand-calculator.

T: The secret number is called "Xan".

First clue

Draw this picture on the board:



T: Xan is in the string picture.

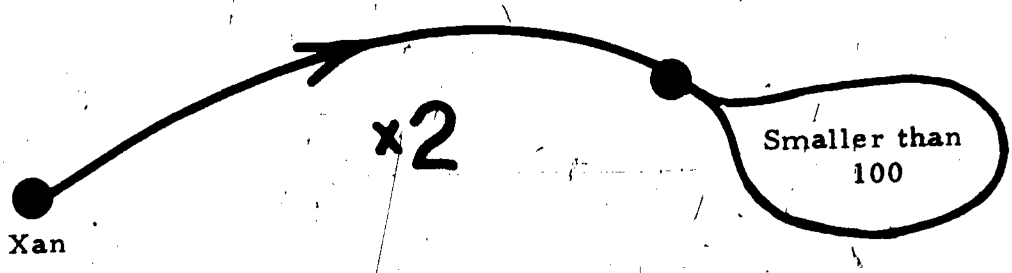
Let the students react to this clue, encouraging them to verbalize the new information that they have about Xan. (Xan is a positive multiple of 3; Xan is an odd number).

Ask them to give many examples of numbers that Xan could be and of numbers that Xan could not be.

The students should conclude that Xan could be 3; 9; 15; 21; . . .
Challenge the students to generate this pattern using the hand-calculator:
$\boxed{3}$ $\boxed{+}$ $\boxed{6}$ $\boxed{=}$ $\boxed{=}$ and so on.

Second clue

Draw this picture on the board:



Let the students play with the clue. As they respond to the clue ask questions such as:

T: Could Xan be 63?

S: No.

T: Why?

S: $2 \times 63 = 126$ which is greater than 100.

T: What is the largest number Xan could be?

S: 45.

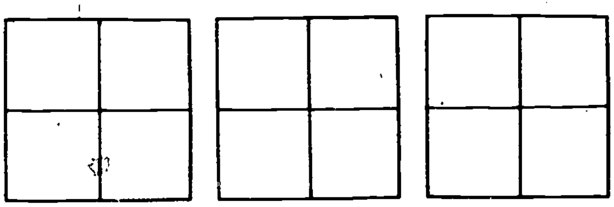
T: What is the smallest number Xan could be?

S: 3.

The students should conclude that Xan is one of the following numbers: 3; 9; 15; 21; 27; 33; 39; 45.

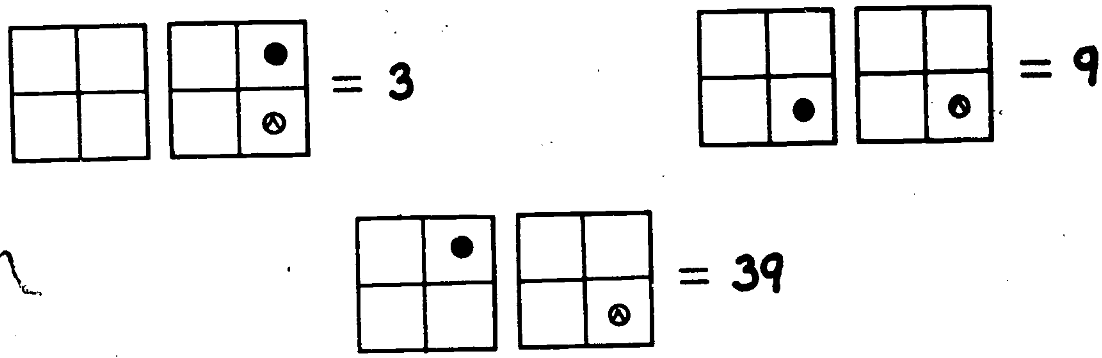
<u>Third clue</u>

Display three Minicomputer boards.



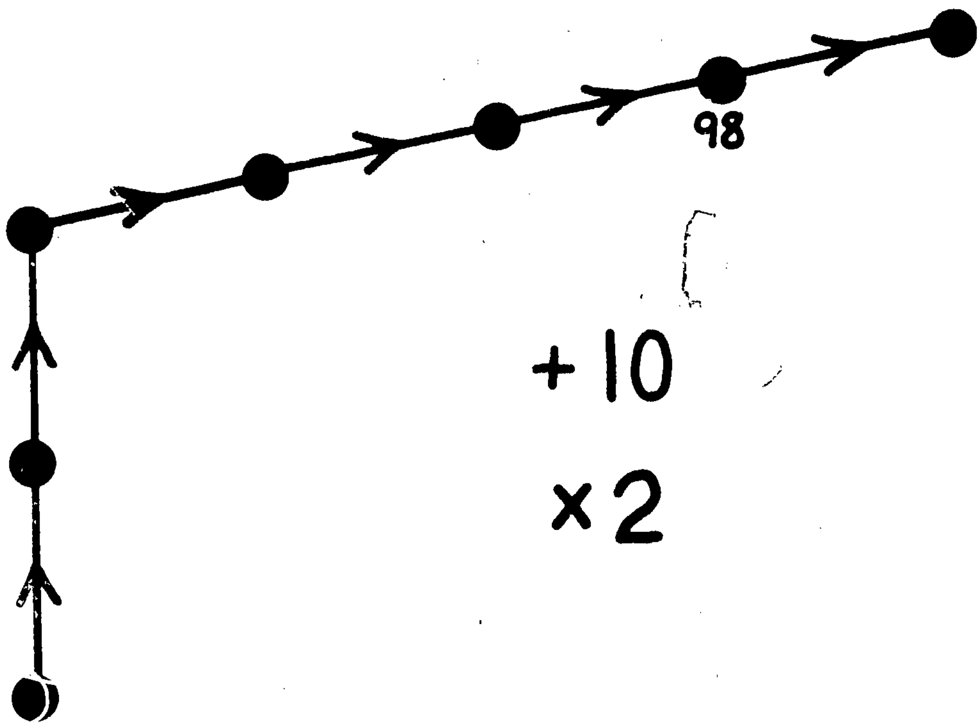
T: Xan can be put on the Minicomputer with exactly one positive checker and one negative checker.

Let the students put the numbers that Xan could be on the Minicomputer. They should conclude that Xan is 3, 9 or 39.



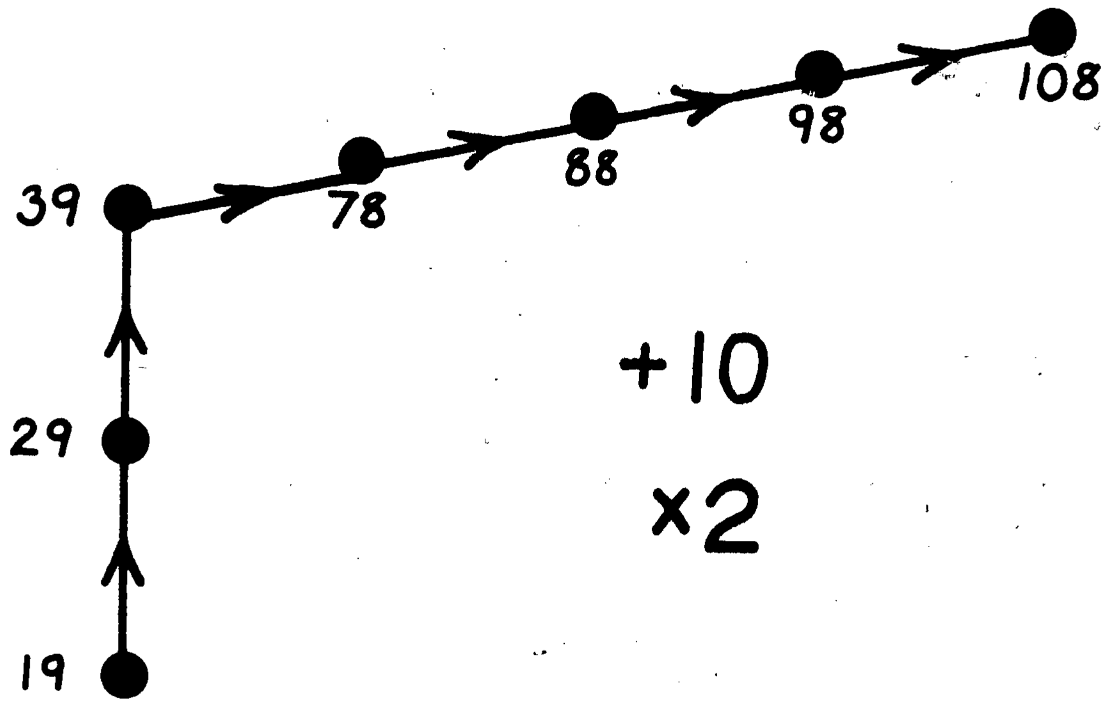
Fourth clue

Draw this picture on the board:



T: Xan is in this picture.

Have the students label the dots :



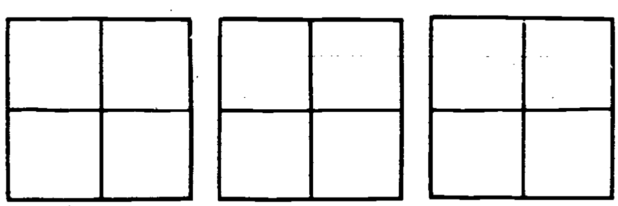
The students should conclude that Xan is 39.

3.12 Who Is Gluck? (with or without the help of a hand-calculator, as you prefer)

T: The name of our secret number is "Gluck".

First clue

Display three Minicomputer boards.



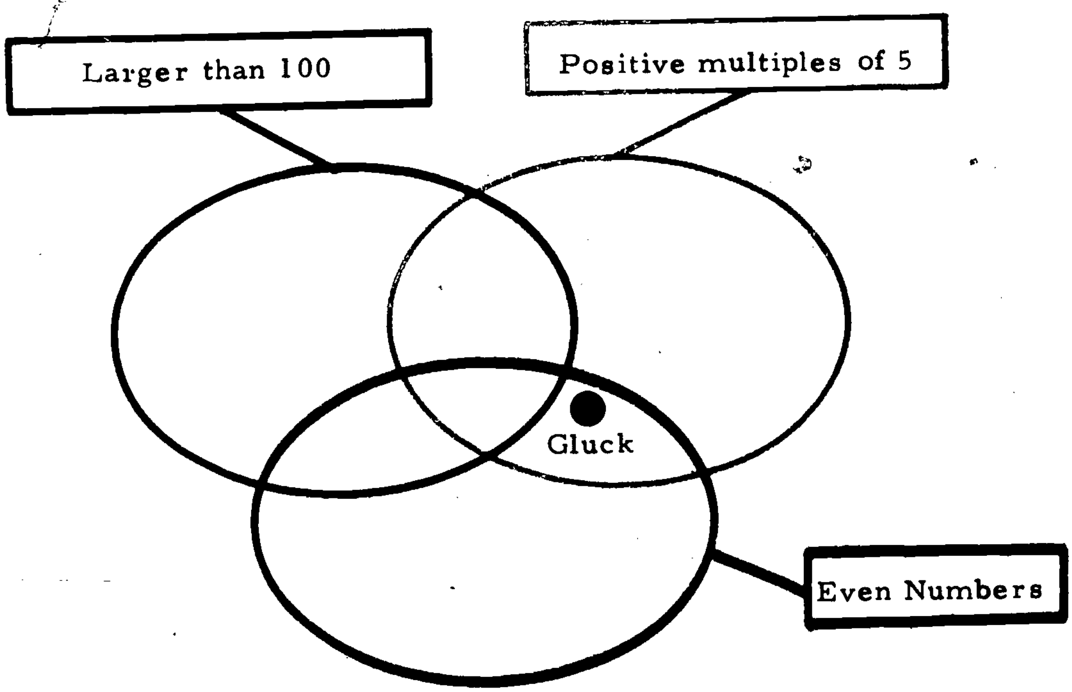
T: Gluck can be put on the Minicomputer with one positive and one negative checker.

Let the students have plenty of time to show numbers that Gluck could be. Obviously the students will not list all of the numbers that Gluck could be. After a while, you may want to ask questions like the following:

T: Could Gluck be 7? (Yes) Show us. 5? (No) 10? (Yes) 99? (Yes) 50? (No) 500? (No) 100? (Yes) 1,000? (No)
Could Gluck be negative? (Yes)
Show us some negative numbers Gluck could not be.
What is the largest number Gluck could be? (799)
What is the smallest? ($^{-}799$)

Second clue

Draw this picture on the board:



T: Gluck is in this string picture.

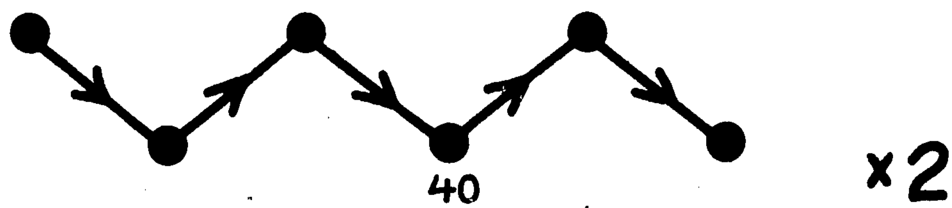
Encourage the students to verbalize the new information about Gluck. (Gluck is not larger than 100; is a positive multiple of 5; is an even number.)

T: Can you name some numbers that Gluck could be?
Can you name some numbers that Gluck could not be?

It is likely that all multiples of 10 up to 100 inclusively will be suggested as numbers that Gluck could be. In this case, remind the students that Gluck can be put on the Minicomputer with one positive and one negative checker. The students should conclude that Gluck is one of the following numbers: 40; 60; 70; 80; 90; 100.

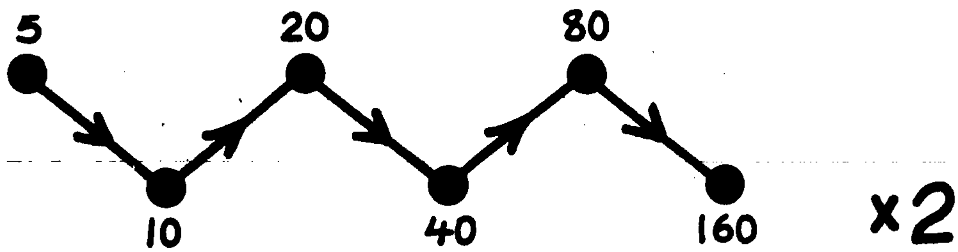
Third clue

Draw this picture on the board:



T: Gluck is in this arrow picture.

Let the students label the dots:



They should conclude that Gluck is 10 or 20 or 40 or 80.

Fourth clue

T: Do you know what a square number is?
Give us some examples of square numbers. ($2 \times 2 = 4$; $6 \times 6 = 36$; $10 \times 10 = 100$; and so on.)
Now the fourth clue: If you add 9 to Gluck, you will get a square number.

After trying each number the students should conclude that Gluck is 40 because $40 + 9 = 49 = 7 \times 7$.

1.13 Who Is Tom?

Give each student a hand-calculator.

T: Tom is our secret number.

First clue

T: Put 968 on the display of your hand-calculator.
Press $\boxed{-}$ $\boxed{5}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ and so on.
Tom will appear on the display.

First let the students play freely with this clue and watch the numbers that appear on the display. Encourage them to express what they notice. After a while, ask some questions. For instance,

T: Could Tom be 902?

S: No. Tom could be 908, 903, 898, . . . but not 902.

T: Could Tom be 983?

S: No. Tom is smaller than 968.

T: Which is the smallest positive number that Tom could be?

S: 3.

T: Could Tom be a negative number?

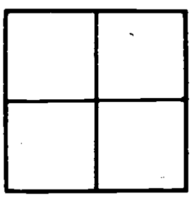
S: Yes.

T: Give some examples of negative numbers that Tom could be.

S: $^{-}2$; $^{-}7$; $^{-}12$; $^{-}17$; $^{-}22$; . . .

<u>Second clue</u>

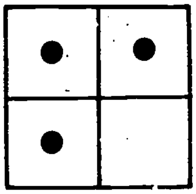
Display one Minicomputer board.



T: Tom can be put on this Minicomputer board using exactly three positive checkers.

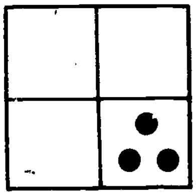
Ask the students to show on the Minicomputer some numbers that Tom could be. It's likely that you will get some wrong answers.

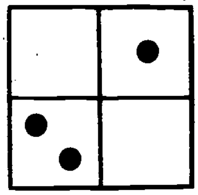
For instance,

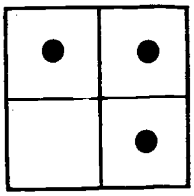
 = 14

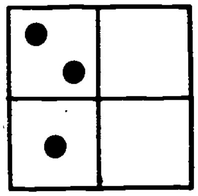
In this case, let the students react and explain that Tom cannot be 14 because "14" ends with "4" and not with "3" or "8".

After some tries, the students should conclude that Tom is one of these numbers :

3 = 

8 = 

13 = 

18 = 

<u>Third clue</u>

T: Put 1,000 on the display of your hand-calculator.
Press $\boxed{-}$ $\boxed{3}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ and so on.
Tom will appear on the display.

Let the students play with this clue for a while and watch the numbers that appear on the display. Challenge them with some questions. For instance, T: If I press $\boxed{=}$ 100 times, what number will appear on the display? Can you answer this question without actually using your hand-calculator?

S: 700 will appear on the display because

$$1,000 - (100 \times 3) = 1,000 - 300 = 700.$$

T: And if I press $\boxed{=}$ 200 times?

S: 400 will appear on the display because

$$1,000 - (200 \times 3) = 1,000 - 600 = 400.$$

T: And if I press $\boxed{=}$ 300 times?

S: 100 will appear on the display because

$$1,000 - (300 \times 3) = 1,000 - 900 = 100.$$

T: Now we know that 100 will appear on the display. Put 100 on the display of your hand-calculator. Hide the display.
Press $\boxed{-}$ $\boxed{3}$ $\boxed{=}$ $\boxed{=}$ . . . (10 times).
What number should be on the display?

S: 70.

T: Look and see if it's correct; then hide the display again.
Press $\boxed{=}$ 10 times again. What number should be on the display?

S: 40.

T: Look and see if it's correct. Then hide the display again. Now press $\boxed{=}$ until one of these numbers appears on the display: 3; 8; 13; 18. Then stop and tell me which is the number on the display. (13 is the only possible answer.)

The students should conclude that Tom is 13.

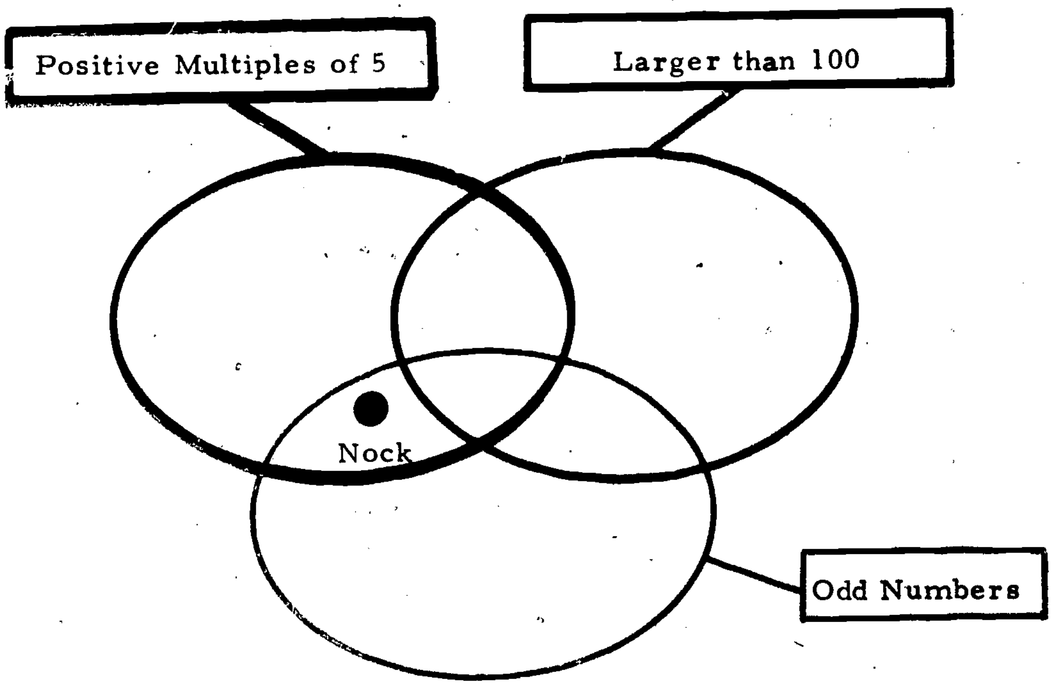
1.14 Who Is Nock?

Give each student a hand-calculator.

T: The name of our secret number is "Nock".

First clue

Draw this picture on the board:



T: Nock is in this string picture.

Let the students play with the clue. Encourage them to verbalize the information they have about Nock. (Nock is a positive multiple of 5, not larger than 100, and odd.)

T: Can you name some numbers that Nock could be?
Can you name some numbers that Nock could not be?

The students should conclude that Nock is one of these numbers: 5; 15; 25; 35; 45; 55; 65; 75; 85; 95.

Second clue

T: Start with 1,000 on the display of your calculator.
Press $\boxed{-}$ $\boxed{3}$ $\boxed{=}$ $\boxed{=}$ and so on.
Nock will appear on the display.

Let the students observe the numbers that appear on the display. Encourage them to verbalize any ideas they have.

T: Start with 1,000 on the display of your calculator. Suppose I press $\boxed{-}$ $\boxed{3}$ and then $\boxed{=}$ 100 times. What number will appear on the display?

S: 700 because $700 = 1,000 - (100 \times 3)$.

T: Try to work this problem without the help of your calculator. Now we have 700 on our display. Suppose I press $\boxed{-}$ $\boxed{3}$ and then $\boxed{=}$ 100 times. What number will appear on the display?

S: 400 because 400 = 700 - (100 x 3).

T: Suppose I press $\boxed{=}$ 100 times more. What number will appear? Try to do this without using your calculator.

S: 100 = 400 - (100 x 3)

T: Now we have 100 on the display of our calculators. Hide the display. Press $\boxed{-}$ $\boxed{3}$ $\boxed{=}$ $\boxed{=}$ until you think 85 is on the display of your calculator. How many times did you press $\boxed{=}$?

S: Five times because 85 = 100 - (5 x 3).

T: What is the next number in our list of numbers for Nock that will appear?

S: 55.

T: How many times did you press $\boxed{=}$?

S: 10 times because 55 = 85 - (10 x 3).

T: What is the next number in our list to appear?

S: 25.

T: How many times did you press $\boxed{=}$?

S: 10 times because 25 = 55 - (10 x 3).

The students should conclude that Nock is 25, 55 or 85.

Third clue

T: Do you remember what a square number is?

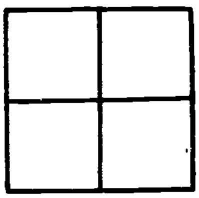
Let the students give examples of some numbers that are square and some that are not square.

T: Now I will give you the third clue. Nock is not a square number.

Let the students work on their own. They should conclude that Nock is 55 or 85.

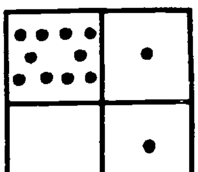
Fourth clue

Display one Minicomputer board.



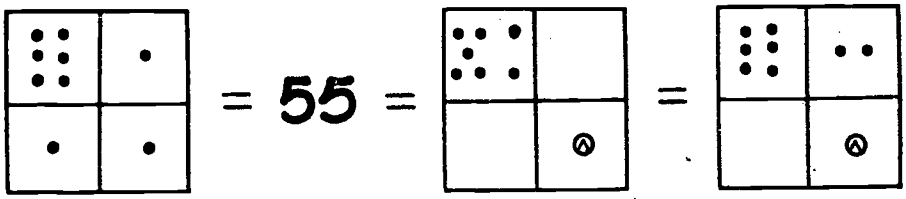
T: Nock can be put on this Minicomputer board with less than 10 checkers (positive or negative).

Ask the students to try to put 55 and 85 on the Minicomputer with less than 10 checkers. After many tries some students will discover that 85 requires at least 12 checkers; i.e.,



= 85

Others will discover that 55 can be represented with 8 or 9 checkers; i.e.,



The students should conclude that Nock is 55.

3.15 Who Is Jig?

Give each student a hand-calculator.

T: The secret number is called "Jig".

First clue

T: Start with 867 on the display of your calculator.
Press $\boxed{-}$ $\boxed{5}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ and so on.
Jig will appear on the display.

Let the students react to this clue and watch the numbers that appear on the display. Encourage them to express their ideas about Jig. Challenge them with some questions. For instance,

T: Could Jig be 793?

S: No. Jig could be 797 or 792 but not 793.

T: Could Jig be 897?

S: No. Jig is smaller than 867.

T: What is the smallest positive number that Jig could be?

S: 2.

T: Could Jig be a negative number?

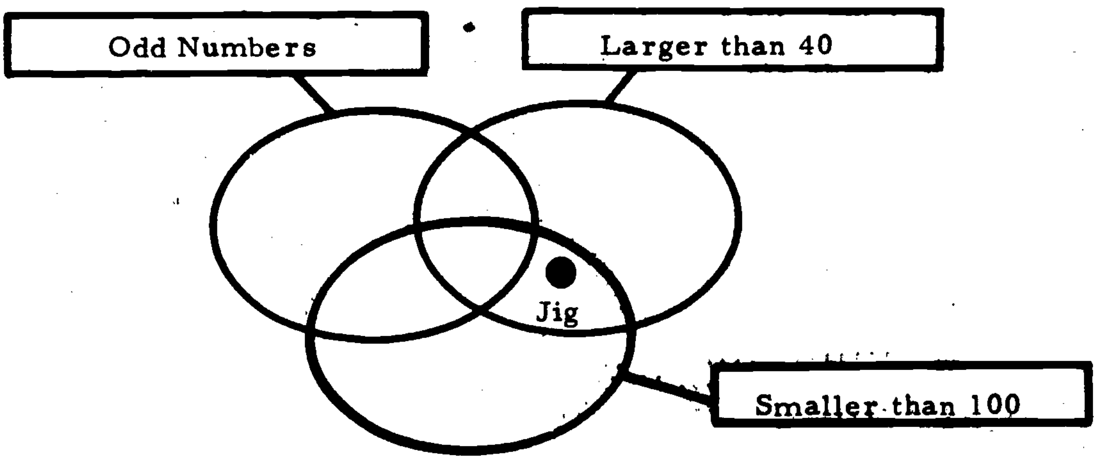
S: Yes.

T: Give some examples of negative numbers that Jig could be.

S: $^{-}3$; $^{-}8$; $^{-}13$; $^{-}18$; . . .

<u>Second clue</u>

Draw the following picture on the board:



T: Jig is in this string picture.

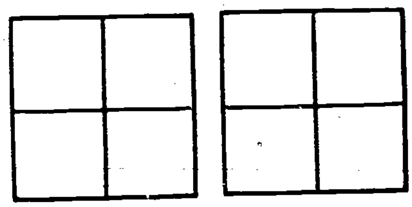
Let the students react to this new clue. Encourage them to verbalize the new information they have about Jig. (Jig is an even number, larger than 40 and smaller than 100.)

T: Can you give examples of numbers that Jig could be? Don't forget our first clue.

After some discussion, the students should conclude that Jig is one of these numbers: 42; 52; 62; 72; 82; 92.

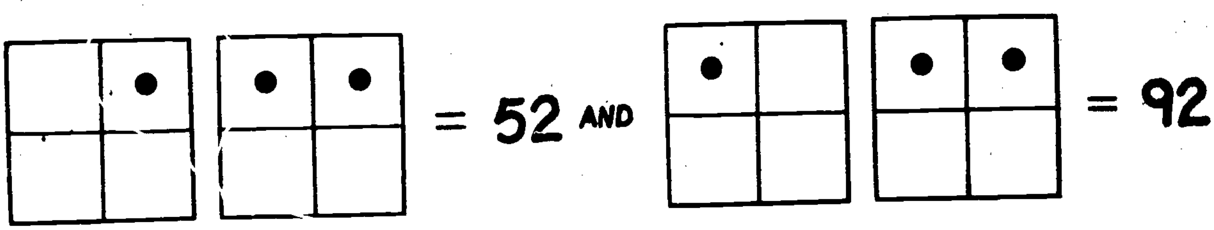
Third clue

Display two Minicomputer boards.



T: Jig can be put on the Minicomputer with one positive checker on the tens' board and two positive checkers on the ones' board.

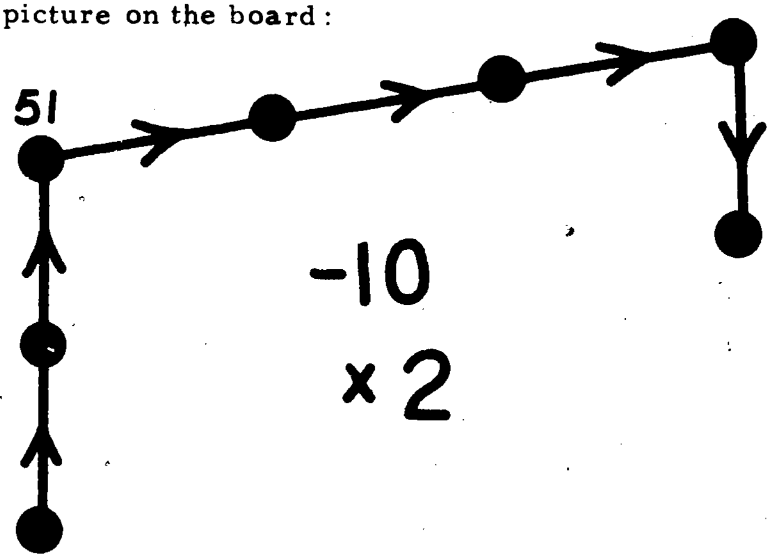
Let the students put the numbers that Jig could be on the Minicomputer. It is likely that 52 or 92 will be overlooked as numbers that Jig could be. Keep in mind that:



The students should conclude that Jig is one of these numbers: 42; 52; 82; 92.

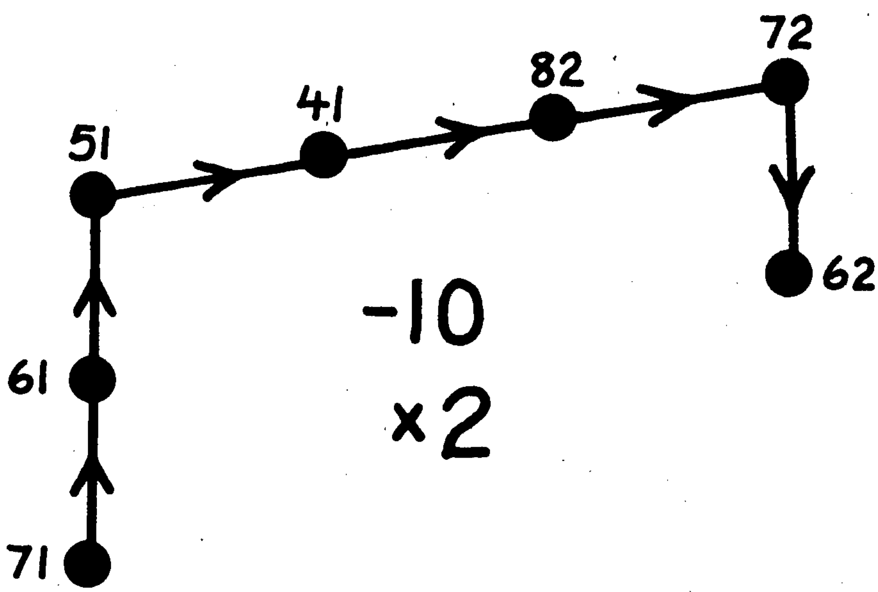
Fourth clue

Draw this picture on the board :



T: Jig is in this arrow picture.

Have the students label the dots :



The students should conclude that Jig is 82.

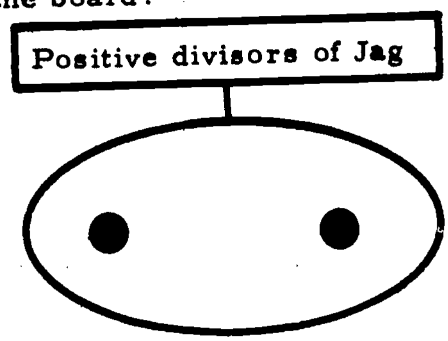
3.16 Who Is Jag?

Give each student a hand-calculator.

T: The secret number is called " Jag ".

First clue

Draw this picture on the board:

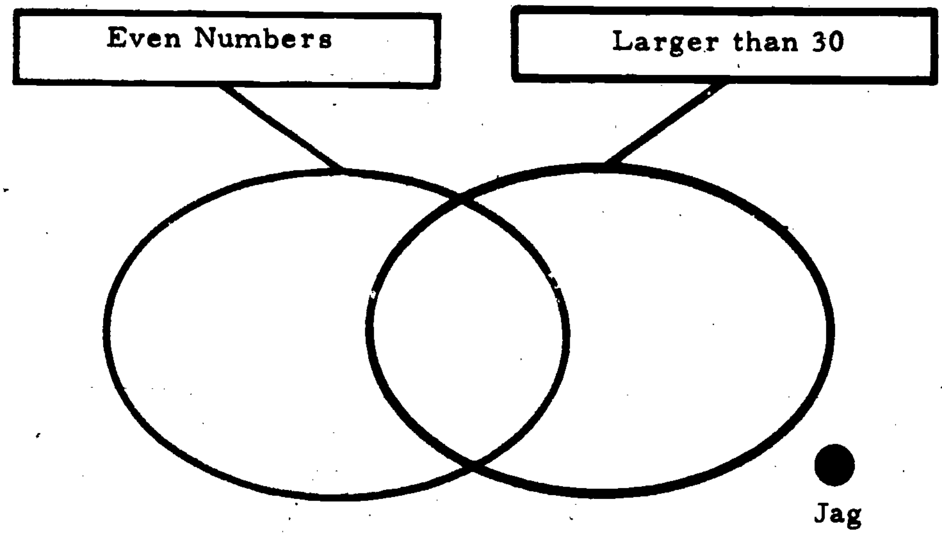


T: Jag is a positive whole number and has exactly two positive divisors. Can you give some examples of numbers that Jag could be and of numbers that Jag could not be?

Give students enough time to explore this clue and give many examples. Jag could be 2, 3, 5, 7, 11, 13, 17, or any prime number.

Second clue

Draw the picture from the top of the next page on the board.



Let the students play with this new clue. Encourage them to verbalize the new information they have about Jag. (Jag is 30 or less than 30, but Jag is odd and hence cannot be 30. Thus, Jag is an odd number less than 30.)

T: Could Jag be 22?

S: No, because 22 is even and Jag is not.

T: Could Jag be 13?

S: Yes.

T: What is the smallest number Jag could be?

S: 3.

T: What is the largest?

S: 29.

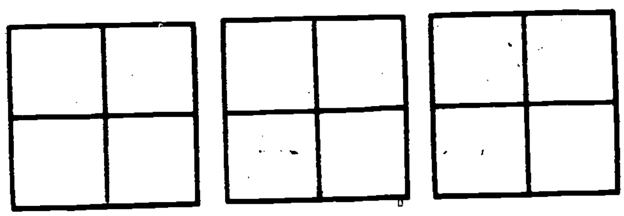
T: What else could Jag be?

S: 3; 5; 7; 11; 13; 17; 19; 23; 29.

Write this list of numbers on the board.

Third clue

Display three Minicomputer boards.



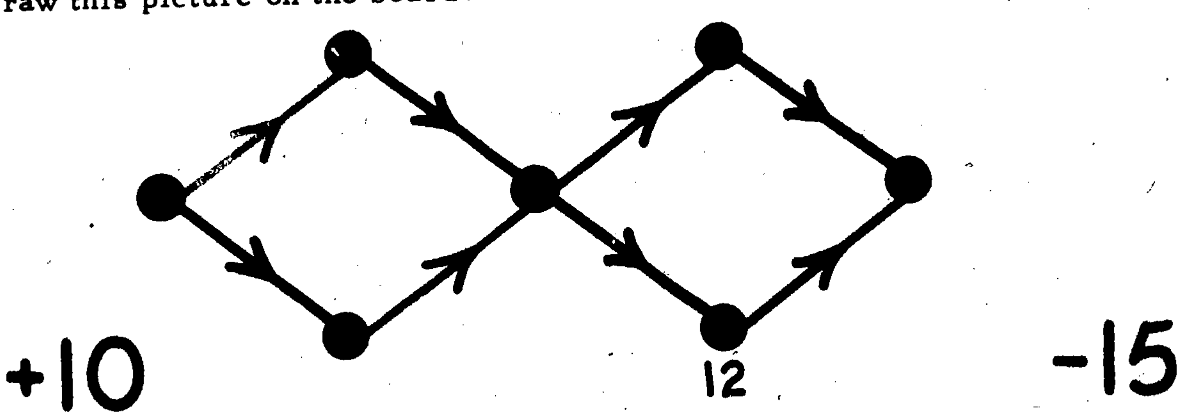
T: Jag <u>cannot</u> be put on the Minicomputer with two checkers (positive or negative).

From the above list, ask the students to display on the Minicomputer the numbers that Jag could not be (3; 5; 7; 11; 19) because they can be put on the Minicomputer using exactly two checkers (positive or negative).

The students should conclude that Jag is 13, 17, 23 or 29.

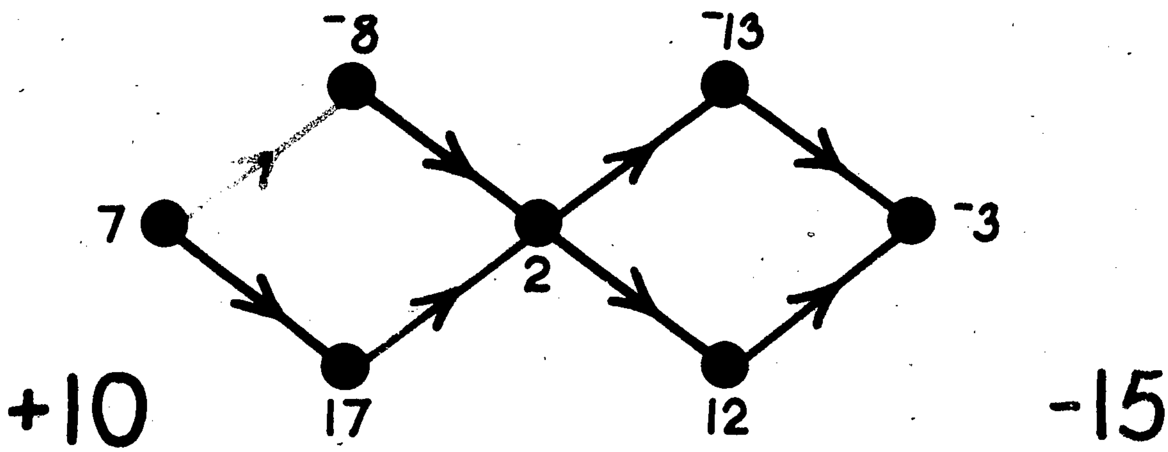
Fourth clue

Draw this picture on the board:



T: Jag is in this arrow picture.

Have the students label the dots:



The students should conclude that Jag is 17.

3.17 Who Is Jug?

Give each student a hand-calculator.

T: The secret number is called "Jug".

First clue

T: Press $\boxed{+}$ $\boxed{3}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ and so on.
Jug will appear on the display.

Let the students react to this clue and watch the numbers that appear on the display. Encourage them to express their ideas about Jug. Challenge them to predict numbers that Jug could be with questions such as:

T: Could Jug be 302? (No); 296? (No); 615? (Yes); 598? (No); 901? (No); 1,000? (No)

Second clue

T: Clear your display and press $\boxed{+}$ $\boxed{4}$ $\boxed{=}$ $\boxed{=}$ and so on. Jug will appear on the display.

Let the students play with this new clue. Continue to encourage them to verbalize their ideas about Jug. Again challenge them to make predictions about Jug.

T: Could Jug be 48?

S: Yes.

T: Could Jug be 16?

S: No. 16 is a multiple of 4 but not of 3.

T: Could Jug be 27?

S: No. 27 is a multiple of 3 but not of 4.

T: What is the smallest number Jug could be?

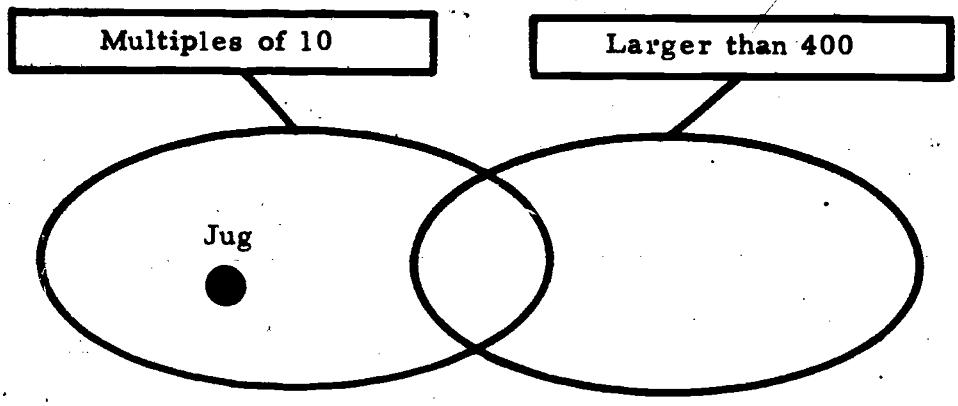
S: 12.

A careful listing of numbers that Jug could be (12 ; 24 ; 36 ; 48 ; . . .) brings to light another pattern. Challenge the students to generate this pattern an easier way using their calculators.

Press $\boxed{+}$ $\boxed{1}$ $\boxed{2}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ and so on.

<u>Third clue</u>

Draw this picture on the board:



T: Jug is in this string picture.

Let the students react to this new clue. Let them verbalize their new ideas about Jug as much as possible. Sometimes a few questions will help. For example,

T: Could Jug be 48?

S: No. 48 is not a multiple of 10.

T: What about 120?

S: Yes.

T: Give some other numbers Jug could be.

S: 60; 240; 360.

T: What is the largest number Jug could be?

S: 360.

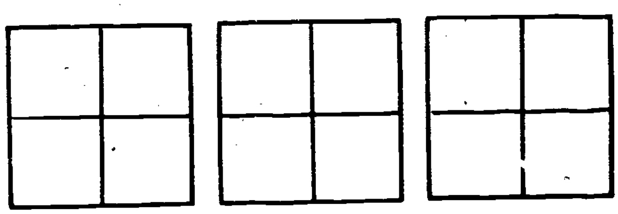
T: And the smallest?

S: 60.

T: Jug is one of these numbers: 60; 120; 180; 240; 300; 360.

<u>Fourth clue</u>

Display three Minicomputer boards.

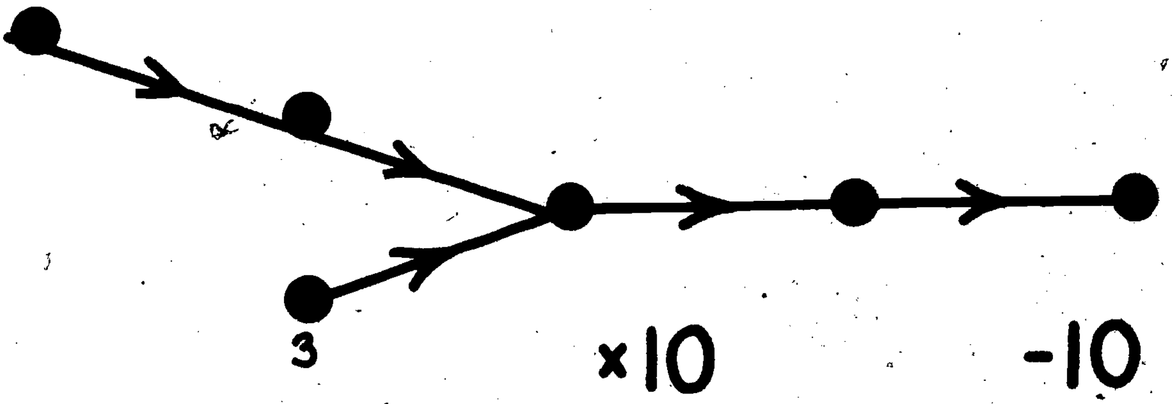


T: Jug can be put on the Minicomputer with exactly three positive checkers on the same square.

Let the students put the numbers that Jug could be on the Minicomputer. The students should find that Jug is 60, 120, 240 or 300.

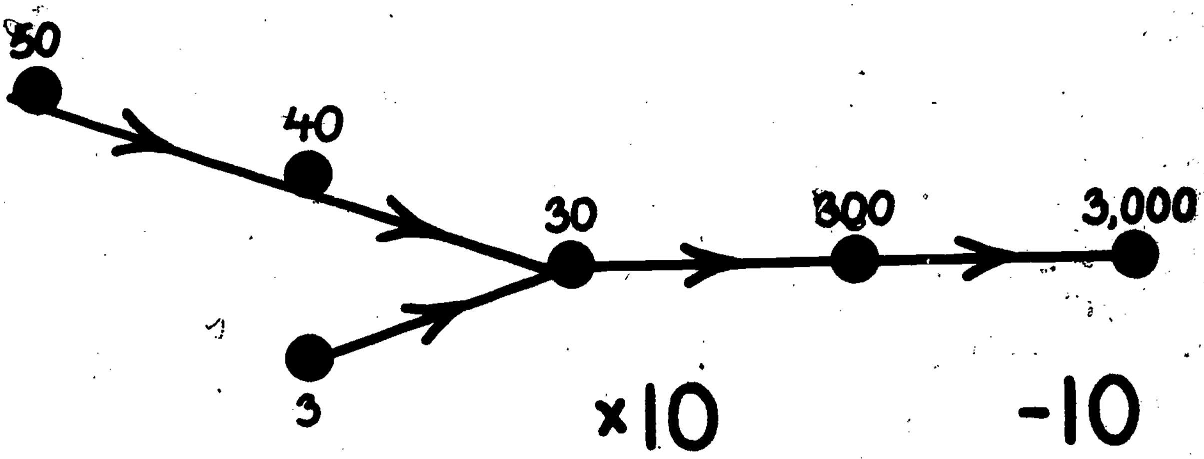
Fifth clue

Draw this picture on the board:



T: Jug is in this arrow picture.

Have the students label the dots:



The students should conclude that Jug is 300.

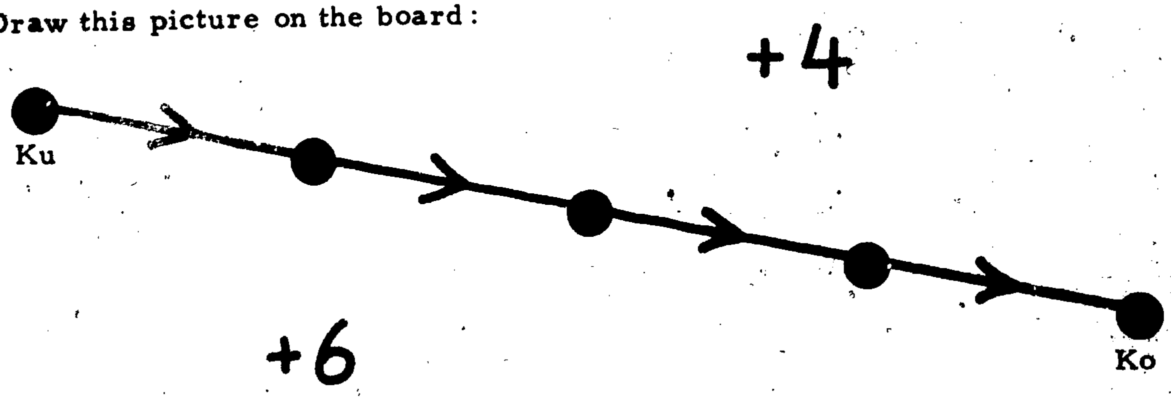
3.18 Who are Ko and Ku?

Give each student a hand-calculator.

T: The secret numbers are called "Ko" and "Ku".

First clue

Draw this picture on the board:



T: Which number is smaller, Ko or Ku?

S: Ku.

T: Ko is how much more than Ku?

S: 20 more.

T: If Ku were 75, what would Ko be?

S: 95.

T: If Ko were 54, what would Ku be?

S: 34.

Use questions of this nature until the pattern is clear.

Second clue

T: Ku is a positive divisor of 36.

Let the students list the positive divisors of 36 on the board. [Answer: 1; 2; 3; 4; 6; 9; 12; 18; 36]

Then recalling the first clue if necessary, let them list all the numbers that Ko could be. [Answer: 21; 22; 23; 24; 26; 29; 32; 38; 56]

Third clue

T: Start with 1,000 on the display of your calculator.
Press $\boxed{-}$ $\boxed{4}$ $\boxed{=}$ $\boxed{=}$ and so on.
Ko will appear on the display.

Let the students explore the patterns that will appear on the display. Encourage them to verbalize any patterns they notice. Then ask:

T: Clear your display and enter 1,000 again. Hide the display and press $\boxed{-}$ $\boxed{4}$ $\boxed{=}$ $\boxed{=}$ . . . ten times.
Who can predict what number will be on the display?

S: 960.

T: Check and see if it's correct. Put the calculators aside. Suppose I start with 1,000 and press $\boxed{-}$ $\boxed{4}$ and then $\boxed{=}$ 100 times. What number will be on the display?

S: 600, because 1,000 - (100 x 4) = 600.

T: Suppose I press $\boxed{=}$ 100 more times. What number will be on the display?

S: 200, because 600 - (100 x 4) = 200.

T: Now we know that 200 will appear on the display. Suppose we start with 200 and press $\boxed{-}$ $\boxed{4}$ $\boxed{=}$ $\boxed{=}$ and so on. Can you give examples of numbers that will appear on the display?

Give students enough time to suggest many numbers.

T: Will 100 appear on the display?

S: Yes, because 200 - (25 x 4) = 100.

T: Will 60 appear on the display?

S: Yes.

T: Will 50 appear on the display?

S: No.

T: Will 0 appear on the display?

S: Yes.

T: Starting from 0, how could we generate all the preceding numbers that appeared on the display?

S: By pressing $\boxed{+}$ $\boxed{4}$ $\boxed{=}$ $\boxed{=}$ and so on.

T: What do you notice about all the numbers?

S: They are multiples of 4.

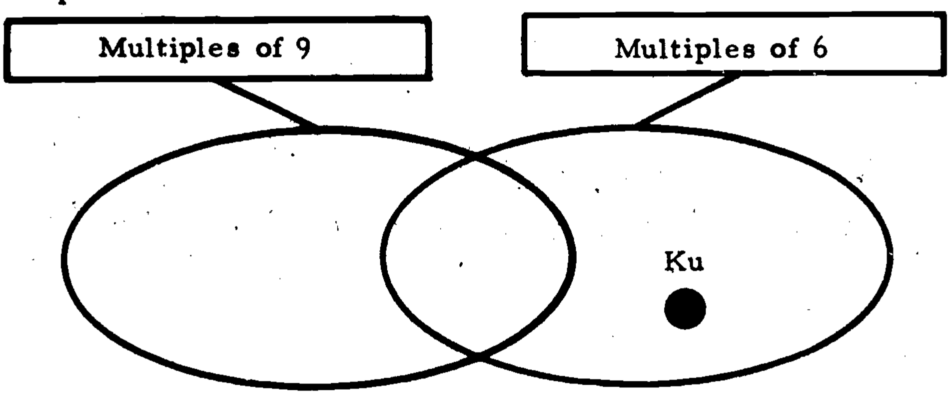
T: Look at the numbers we have listed for Ko. (21; 22; 23; 24; 26; 29; 32; 38; 56)
Which of them are multiples of 4 and will appear on the display?

S: 24; 32; 56

The students should conclude that Ko is 24, 32 or 56 and that Ku is 4, 12 or 36.

Fourth clue

Draw this picture on the board:



T: Ku is in this string picture.

Let the students react to this clue. Encourage them to discuss the new information they have about Ku. (Ku is a multiple of 6; Ku is not a multiple of 9.) The students should conclude that Ku is 12 and Ko is 32.